ESSAYS ON BANK OPTIMAL PORTFOLIO CHOICE UNDER LIQUIDITY CONSTRAINT

A Dissertation

by

EUL JIN KIM

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2012

Major Subject: Economics

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Co-Chairs of Committee,	Hwagyun Kim
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ABSTRACT

Essays on Bank Optimal Portfolio Choice under Liquidity Constraint.

(August 2012)

Eul Jin Kim, B.A., Seoul National University

Co–Chairs of Advisory Committee: Dr. Hwagyun Kim
Dr. Ryo Jinnai

Long term asset creates more revenue, however it is riskier in a liquidity sense. Our question is: How does a liquidity constrained bank make decisions between profitability and liquidity? We present a computable DSGE model of banks optimal portfolio choices under liquidity constraints. Our theory predicts that liquidation plays an important role in a bank's portfolio model. Even though liquidation is an off-equilibrium phenomenon, banks can have rich loan portfolios due to the possibility of liquidation. Liquidity condition is a key factor in banks portfolio. In a moderate liquidity situation, a bank can lend more profitable longer term loans, however, if a shock in liquidity is expected, then the bank lends more loans in short term. According to the liquidity conditions, the bank can have medium term loans which is different from other previous literature.

In addition, we extend our model to the bank's securities business where the bank's debts are largely short term deposit. Our theory predicts that the bank securities business produces a chasm between a real liquidity of economy and market liquidity. Banks can have more liquidity by selling their securitized loans, and as our model already pointed out, a good liquidity condition makes the bank have more profitable but less liquid long term loans. As a consequence, long term loans are accumulated with this securitization, simply because a long term loan gives higher revenue. Any market turbulence can invoke a problem in economy wide liquidity.

To my parents, my wife and my children

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I am happy to have this opportunity to express my gratitude to those who helped to make this dissertation possible. Without these people's love, support and dedication, I might not have been able to finish this long and demanding journey of doctoral studies.

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I would like to thank my parents, late Kiyul Kim and Eunsoon Park. They have always supported me. My parents-in-law, Youngsik Kim and Sukja Jung, have encouraged me with their best wishes. Finally, I want to thank my wife, Soyeun Kim and my son Chan Woo Kim and daughter Jihyo Kim. They have always been there to cheer me up and to stand by me through the good times and the bad.

TABLE OF CONTENTS

CHAPTER Page

LIST OF TABLES

LIST OF FIGURES

FIGURE Page

CHAPTER I

INTRODUCTION

Banks finance short-term from the public and invest to project holders considerably long-term. The miss-match of maturities between banks liabilities and assets is a good topic of many researchers and also a real problem of bank managers. Although a bank should be ready to pay her debts without any prior notice, she can have long-term assets in that, first, she has 'the law of large numbers' and, second, she can make new debts as Diamond and Dybvig (1983) argued. Banks can make estimations of cash-in and -out flows due to their long experiences and/or well established statistical methods. This is 'the law', the well known and easy to understand. However, it is not quite simple to catch the meaning of the argument, when they said that making new debts to pay old debts is one of key functions of banking business. Anybody having a highly profitable and strongly promissible project can find lenders to invest, if she has proper creditworthiness. Banks concern their reputations with the same reason. They explained that, since the bank has a specialized skill to find where the profitable projects are, and who owns, and since the bank always tries to keep good records to survive, a new debt to the bank means that the economy does not lose a new profit opportunity. Therefore making new debts to pay old debts means the economy can enjoy more fruits from longer term projects. This can be accomplished the most efficiently only by the bank. Their arguments, and Diamond and Rajan (2001) analogous extension provide a rich insightfulness to understand the core aspects of banking business. Banks make liquid deposits with illiquid loans, and

This dissertation follows the style of *Econometrica*.

therefore profits are made and distributed from borrowers to lenders.

Now, more practical problems of 'how much' and 'how long' need to be answered. There are two strands of literature, bank portfolio and loan maturity. First, a strong tradition leans on the linear programming, such as asset and liability management (ALM) and bond portfolio management literatures. ALM models deal with cash flow, liquidity, risk and return of bank's assets and liabilities. Based on Markowitz (1959) portfolio selection theory, Pyle (1971) provided a static model, and Kallberg and Ziemba (1983) extended to a dynamic model. Another ALM approach using stochastic dynamic programming is found by Eppen and Fama (1968, 1969, 1971), and Kusy and Ziemba (1986). ALM model was once so popular, but basically too technical and lacks microfoundations. A bond portfolio model handles banks assets and liabilities management. Crane (1971) constructed a discrete stochastic multi-stage model and found that optimal portfolio is one of (1) shortest only, (2) longest only or (3) mix of the two, so medium term bonds cannot be an optimal. Furfine (2001) analyzed the bank portfolio changes according to the Basel Capital Accord. He found that banking regulation encourages banks to shift from loans to safer assets. Second, loan maturity problem is handled by Coleman, Esho, and Sharpe (2002), Gottesman and Roberts (2004) and Fedorenko, Schafer, and Talavera (2007). Coleman, Esho, and Sharpe (2002) argued that bank monitoring ability, bargaining power, risk and syndicate structure significantly affect the loan maturity and pricing. Gottesman and Roberts (2004) found evidences supporting both the tradeoff, which is lenders are compensated for longer maturity loans, and the credit quality hypotheses, that is lenders limit their exposure by forcing riskier borrowers to take short term loans. Fedorenko, Schafer, and Talavera (2007) established an empirical model of Diamond (1991), and found (1) the best and the worst rated loans tend to have shorter maturities than loans with an intermediate rating (consistent with Diamond), and (2) a negative association between ratings and the maturity of the loans to sole proprietors (differ from Diamond and other empirical literature).

We provide a model to handle directly a liquidity constrained bank's optimal loan portfolio choice, and this is the first DSGE approach dealing with this problem, as far as we know. We borrow major concepts from the Diamond and Rajan (2001) paper: relationship lending, specialized skills in loan collection, renegotiation, and liquidation process. In addition, we introduce an incentive constraint scheme against bank runs, assumptions on investment- and production-technologies to standardize loan contracts. As a result, we have effectively established a DSGE model with infinite horizon and multiple loans.

The incentive constraint makes the bank solvent with the demand deposits at any time and states, so the bank creates new liquidity. This constraint can rule out the bank run in equilibrium path, however, it causes the bank not to avoid their due diligence. Therefore there is no financial frictions of information asymmetry, and moral hazards. For the standardization, we allow *arbitrage* between projects with different technologies. For instance, a less profitable long term project will be dominated by more profitable short term projects. More technically, we assume an efficient investment interval of technology, which can be thought of as a minimized input for the same output. With these two moderate assumptions, we got a simple function representation of bank's loan portfolio on $R_+ \times [0, 1]$ plane. The computational burden dramatically reduced.

Our theory predicts that liquidation plays an important role in the bank's portfolio model. Even though liquidation is an off-equilibrium phenomenon, banks can have rich loan portfolio due to the possibility of liquidation. Liquidity condition is a key factor in banks portfolio. In a moderate situation, a bank can lend more profitable longer term loans, however, if a shock in liquidity is expected, then the bank lends more loans in short term. According to the liquidity conditions, the bank can have medium term loans which is different from Crane (1971).

Banking business is quite different from their traditional aspects. They have many securities and trading assets in their balance sheets. Without the securitization, we could not explain nowadays banking business properly. Securitization in banking business makes better off our economy in good seasons definitely. However, with the depressed securities market, banks can deepen the business cycles. Bank's debts are largely short term deposit. Our theory predicts that the bank securities business produces a chasm between a real liquidity of economy and market liquidity. Banks can have more liquidity by selling their securitized loans, and as our model already pointed out, a good liquidity condition makes the bank more profitable but less liquid long term loans. As a consequence, long term loans are accumulated and every participant is happy with this securitization, simply because a long term loan gives higher revenue. However, the economy has less liquidity in short term. Any market turbulence can invoke a problem in economy wide liquidity.

As Rajan (2006) pointed out, banking business has changed to more risky areas, due to technical change, deregulation and institutional change. Banks improve their risk management abilities and risk taking capacities. This trend is an inevitable consequence of severe competitions. The above three factors have pushed up the banks for more profitability to survive. Banks have to involve in high profit business like securities, derivatives and exotic new financial products. Banking business is quite different from their traditional aspects. They have many securities and trading assets in their balance sheets. Without the securitization, we could not explain nowadays banking business properly. Securitization in banking business makes better off our economy in good seasons definitely. However, with the depressed securities market, banks can deepen the business cycles. Banks' debts are largely short term deposit.

Many literatures analyze the securities business in banking, however, very few directly connecting the securities to the bank's short term liabilities. We provide a useful tool to analyze bank's traditional loan and deposit business connected with new securities business.

Recent sub-prime mortgage crisis is believed that banks misalignments of resources is a main cause. There are many literatures analyzing this crisis with financial frictions(Gertler, Kiyotaki, and Queralto (2010)). Our model provides why banks should involve in securities business, and how crisis evolves inherently in the bank's securities business.

Our theory predicts that the bank securities business makes a chasm between a real liquidity of economy and market liquidity. Banks can have more liquidity by selling their securitized loans, and as our previous model already pointed out, a good liquidity condition push the bank more profitable but less liquid long term loans. As a consequence, long-term loans are accumulated, and every participant happy with this securitization, simply because long term loan gives higher revenue. However, the economy has less liquidity in short term. Any market turbulence can invoke a problem in economy wide liquidity. Therefore bank regulators and macroeconomic policy agencies should make a brake to the securities business of banks. We suppose a liquidity taxation in banks' securities business as a provision for the shortage of liquidity and government direct money injections to the market as a crisis response.

CHAPTER II

BANK OPTIMAL PORTFOLIO CHOICE UNDER LIQUIDITY CONSTRAINT

A. Introduction

Banks finance short-term from the public and invest to project holders considerably long-term. The miss-match of maturities between banks liabilities and assets is a good topic of many researchers and also a real problem of bank managers. Although a bank should be ready to pay her debts without any prior notice, she can have long-term assets in that, first, she has 'the law of large numbers' and, second, she can make new debts as Diamond and Dybvig (1983) argued. Banks can make estimations of cash-in and -out flows due to their long experiences and/or well established statistical methods. This is 'the law', the well known and easy to understand. However, it is not quite simple to catch the meaning of the argument, when they said that making new debts to pay old debts is one of key functions of banking business. Anybody having a highly profitable and strongly promissible project can find lenders to invest, if she has proper creditworthiness. Banks concern their reputations with the same reason. They explained that, since the bank has a specialized skill to find where the profitable projects are, and who owns, and since the bank always tries to keep good records to survive, a new debt to the bank means that the economy does not lose a new profit opportunity. Therefore making new debts to pay old debts means the economy can enjoy more fruits from longer term projects. This can be accomplished the most efficiently only by the bank. Their arguments, and Diamond and Rajan (2001) analogous extension provide a rich insightfulness to understand the core aspects of banking business. Banks make liquid deposits with illiquid loans, and therefore profits are made and distributed from borrowers to lenders.

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The next section describes model features. Section C gives us some characterizations and the simple function representation, and the following section presents some calibration strategies, summary of the results. Then we conclude.

B. Model

1. Environment

The model exists on the discrete and infinite time space. There are three aggregate shocks: a total factor productivity (TFP) shock, $A_t \in \mathcal{A} \equiv \{A_1, A_2, \cdots, A_l\}$, and two shocks that determine the distribution of capital quality $\phi_t \in \Phi \equiv \{\phi_1, \phi_2, \cdots, \phi_m\}$, and that of time preference $g_t \in \mathcal{G} \equiv \{g_1, g_2, \cdots, g_n\}$. (A_t, ϕ_t, g_t) are mutually independent, and form a joint Markov process that evolves according to a transition probability $\Gamma : (\mathcal{A} \times \Phi \times \mathcal{G}) \times (\mathcal{A} \times \Phi \times \mathcal{G}) \rightarrow [0, 1]$ with the standard assumptions. There exist two goods: a perishable consumption (final) goods and a capital goods.

a. Agents

There are three types of agents: producers (final goods producers and a representative capital goods producer), households and a representative bank. Producers are members of one big family. They share all earnings, and eat completely the whole profits at the time they earn. Since they are assumed to have no capitals, they should borrow from the banker to run their businesses. Households supply labor services to the producers inelastically, and they have some initial endowments. They can consume and save the labor income. Since the banker has her own specialized skills in relationship lending and loan collection, she can efficiently intermediate between the borrowers and the savers.

b. Capital Quality, Capital Unit and Project (or Capital)

We introduce a capital unit k with its quality ω, according to Bigio (2011). At each time period, capital K held by the final goods producer is divisible into a continuum of units, and each unit is identified by its quality $\omega \in \Omega \equiv [0,1]$. We call an integrated mass of the capital units a *project* or a *capital*, $K_t^{\omega_0} = \int_0^{\omega_0} k_t(\omega)d\omega, \omega_0 \in \Omega$. The capital cannot be directly inputted into a production process without a capital quality adjustment shock $f_{\phi_t}(\omega)$ at each period. So, the multiplicative shock $f_{\phi_t}(\omega)$ which determined by the realization of ϕ_t, can be regarded as a homogenizing process of the different quality capital units. In addition, there is a fixed increasing differentiable function $\lambda(\omega) : [0,1] \to [0,1]$, which can be interpreted as a residual proportion of capital unit after production (one minus depreciation rate). By the definition of $\lambda(\omega)$, we can understand the capital quality ω more clearly. Durable capital units such as machines, buildings have high capital quality, while parts, components, and fuel have $'0'$ or low quality. Therefore, we can assume that each project is a mass of capital units on the interval of capital quality Ω, starting from zero $[0, \omega_0]$.

c. Investment Technology and Production Periods

The investment technology transforms the consumption goods into the following period capital goods. For the transformation cost, there is a fixed increasing differentiable function $q_t(\omega)$ of capital quality, and the cost is $Q_t^{\omega_0} = \int_0^{\omega_0} q_t(\omega)d\omega, \omega_0 \in \Omega$. Therefore to install a project of $K_t^{\omega_0}$ amount, the producer needs $I_t = Q_t^{\omega_0} K_t^{\omega_0}$ amount of consumption goods investment. We assume that the capital goods are produced with a rectangular shape on the domain of capital quality Ω. This assumption is not critical to our model results if the shape can be represented by any functional forms, but it can make the model developments easy. We suppose the final producer invests I_t at time t. At the beginning of time $t+1$, she owns $K_t^{\omega_0} = \frac{I_t}{Q_t^{\omega_0}} \equiv \int_0^{\omega_0} k_t(\omega_0) d\omega, k_t(\omega_0) \in \mathbb{R}_+$ amount of capital. After the distributional adjustment shock, the amount of $K_t = \int_0^{\omega_0} f_{\phi_t}(\omega) k_t(\omega_0) d\omega$ capital is in use in production process. Noticed that the superscript ω_0 doesn't be used in K_t after the adjustment shock. After the production process, the capital discomposes into each capital unit, and the remains be $K_t^{\omega_0} = \int_0^{\omega_0} f_{\phi_t}(\omega)\lambda(\omega) k_t(\omega_0) d\omega$. And we define the number of production periods any project or capital can be engaged in as follows,

Definition II.1. Any project has a production periods $N(\omega_0, \eta, \phi_t) \geq 0$ such that $N(\omega_0, \eta, \phi_t) = \arg\max_n \{\int_0^{\omega_0} \mathbb{E}_{\phi_t}[\prod_{k=1}^{n} f_{\phi_{t+k}}(\omega)]\lambda(\omega)^n k_{t+1}(\omega_0) d\omega > \eta$, where $\int_0^{\omega_0} k_{t+1}(\omega_0) d\omega = 1, \eta < 1\}$, where $\mathbb{E}_{\phi_t}[f_{\phi_{t+1}}(\omega) f_{\phi_{t+2}}(\omega)] = \sum_{\phi_i, \phi_j \in \Phi} f_{\phi_j}(\omega)\mathbb{P}(\phi_j \mid \phi_i) f_{\phi_i}(\omega)\mathbb{P}(\phi_i \mid \phi_t)$..

The definition means that after some number of production periods the remaining capital should exceed a minimum amount of capital η given a unit of capital investment, to be engaged in the following period's production process. Since the periods depend on the interval of capital quality $[0, \omega_0]$, and given information η and ϕ_t, we can fix the periods of a project at the time of investment. The project with high quality has longer production periods by the definition. Moreover, we denote an interval of capital qualities with same production periods $\omega_t(n) = \{\omega : N(\omega_0, \eta, \phi_t) = n\}$, and denote the element of the interval $\omega_t^n \in \omega_t(n)$.

d. Production Technology and Project Arbitrage

The final goods are produced by the following production technology,

$$Y_t = A_t K_t^{\alpha}, 0 < \alpha < 1,$$

and assume as usual that producers get the proportion α of the output, and the others $(1-\alpha)$ goes to the labor income. Then the steam of producers' expected total revenues will be

$$(R_{t+1},\cdots,R_{t+n})=\{\alpha\mathbb{E}_{A_t}(A_{t+j})\int_0^{\omega_t^n}\mathbb{E}_{\phi_t}[\prod_{k=1}^{j}f_{\phi_{t+k}}(\omega)]\lambda(\omega)^{j-1}k_{t+1}(\omega_t^n)d\omega\}_{j=1}^n,$$

where $\mathbb{E}_{A_t}(A_{t+2})=\sum_{A_i,A_j\in\mathcal{A}}A_i\mathbb{P}(A_i\mid A_j)\mathbb{P}(A_j\mid A_t)$.

We assume that the expected revenues are decreasing as capital depreciates, $R_{t+1}\geq R_{t+2}\geq\cdots\geq R_{t+n}$, and it is not natural that bigger output can be expected with quite smaller capital input. Then we can consider the arbitrage between projects. Let's suppose we can have two different projects $K_{t+1}^{\omega_t^m}, K_{t+1}^{\omega_t^n}$, with the same amount of consumption goods investment. We allow that the inferior projects are wiped out by the superior projects with respect to the efficiencies of investment and production technology. The arbitrage conditions are as follows, (1) Without loss of generality $m\leq n$, for each period before short term project matures the bigger revenue producing project dominates the smaller revenue producing project. Formally if $^\forall k\leq m$, $R_{t+k}^{\omega_t^m}\geq R_{t+k}^{\omega_t^n}$, project $K_{t+1}^{\omega_t^m}$ dominates project $K_{t+1}^{\omega_t^n}$, and if $^\forall k\leq m$, $R_{t+k}^{\omega_t^m}\leq R_{t+k}^{\omega_t^n}$, project $K_{t+1}^{\omega_t^m}$ is dominated by project $K_{t+1}^{\omega_t^n}$. (2) Without loss of generality $m\leq n$, if a project producing the bigger discounted sum of revenues upto the short term project maturity dominates the other project producing the smaller discounted sum of revenues. Formally, if $\sum_k^m\Lambda^kR_{t+k}^{\omega_t^m}\geq\sum_k^m\Lambda^kR_{t+k}^{\omega_t^n}$, then project $K_{t+1}^{\omega_t^m}$ dominates project $K_{t+1}^{\omega_t^n}$, and if $\sum_k^m\Lambda^kR_{t+k}^{\omega_t^m}\leq\sum_k^m\Lambda^kR_{t+k}^{\omega_t^n}$, then project $K_{t+1}^{\omega_t^m}$ is dominated by project $K_{t+1}^{\omega_t^n}$, where Λ is a proper discount rate. After the arbitrages in projects, survival projects should have the following properties, (1) the shorter project must have higher liquidity than longer project, (2) the longer project must have bigger profits than the shorter project. We formally assume the properties,

Assumption II.2. The parameter A_t, functions $f_{\phi_t}(\omega), \lambda(\omega), q(\omega)$ and transition probability Γ are well arranged, so that

(a) ${}^\forall t, {}^\forall A_t \times \phi_t \in (\mathcal{A} \times \Phi)$, $R_{t+1} \geq R_{t+2} \geq \cdots \geq R_{t+n}$, and

(b) there exists a sufficient number of projects after the project arbitrages.

e. Loan Contract and Liquidation

Our loan markets are designed following the Diamond and Rajan (2001). A relationship lender knows where the best and the alternative technology locate in, and there will be renegotiation process after the states realizes. If the banker refuses the producer's offer, liquidation will take place. Our model has a big difference in that the loan contracts involve loan repayment schedules, since our focus is on the different maturity loan contracts and their liquidity effects. We assume that the number $'n'$ is sufficiently large so that we can analyze a bank's loan portfolio. Loan contracts can be characterized by the following quadruple $(a_{t+1}^{(n)}, n, \{P_{t+j}\}_{j=1}^n, \{m_{t+j}\}_{j=1}^n)$,where $a_{t+1}^{(n)}$ indicates the amount of loan made at time t, n the maturity of the loan, P_{t+j} the promised amount of repayment at time $t+j$, m_{t+j} the liquidation value from the remaining capitals at the beginning of time $t+j$. We suppose that the banker's collecting skill be represented by a parameter γ. Then

$$\begin{aligned} P_{t+j} &= \gamma\alpha\mathbb{E}_{A_t}(A_{t+j})\{\int_0^{\omega_t^n} \mathbb{E}_{\phi_t}[\prod_{i=1}^{j} f_{\phi_{t+i}}(\omega)]\lambda(\omega)^{j-1}k_{t+1}(\omega_t^n)d\omega\}^\alpha, \\ m_{t+j} &= \gamma\int_0^{\omega_t^n} \mathbb{E}_{\phi_t}[\prod_{i=1}^{j} f_{\phi_{t+i}}(\omega)]\lambda(\omega)^{j-1}k_{t+1}(\omega_t^n)d\omega, \end{aligned}$$

where $\int_0^{\omega_t^n} q_t(\omega)d\omega \int_0^{\omega_t^n} k_{t+1}(\omega_t^n)d\omega = a_{t+1}^{(n)}$. If the banker accepts the renegotiation offer from the producer, the banker gets a real delivery

$$r_{t+j} = \gamma\alpha A_{t+j}\{\int_0^{\omega_t^n} [\prod_{i=1}^{j} f_{\phi_{t+i}}(\omega)]\lambda(\omega)^{j-1}k_{t+1}(\omega_t^n)d\omega\}^\alpha.$$

Therefore, the necessary conditions for loan liquidation is (1) the realized revenue is smaller than the promised repayment, $r_{t+k} < P_{t+k}$, (2) the liquidation value is bigger than the current period's revenue, $m_{t+k} > r_{t+k}$. Denote $a_{t+1}^{(j,n)}, 0 \leq j \leq n$, a loan contract liquidated at time $t+j$ with maturity n,and $j = 0$ denotes no liquidation of the contract. Then we can describe the liquidation possible area. Denote $\mathcal{U}^{(j,\cdot)} \equiv \mathcal{A} \times \Phi^j$, and $\Psi_{t+1}^{(j,n)} \equiv \{\psi_{t+1}^{(j,n)} \in \mathcal{U}^{(j,\cdot)} : r_{t+j} < P_{t+j}$, and $r_{t+j} < m_{t+j}$, given $a_{t+1}^{(j,n)}, A_t, \phi_t\}$. And note $\mathbb{P}(\Psi_{t+1}^{(j,n)})$ the probability of the liquidation possibility area.

f. Efficient Interval of Capital Quality

Given the states A_t, ϕ_t, and the amount of loan $(a_{t+1}^{(\cdot,n)})$ with loan maturity n, there exists a unique capital quality interval $[0, \omega_{t+1}^{(\cdot,n)*}], \omega_{t+1}^{(\cdot,n)*} \in \omega_{t+1}(n)$ to achieve the maximum output, and we define the optimal interval $[0, \omega_{t+1}^{(\cdot,n)*}]$ which ensures the efficient investment.

$$\omega_{t+1}^{(\cdot,n)*} = \arg \max_{\omega_{t+1}^n \in \omega_{t+1}(n)} [\sum_{i=1}^{n} \mathbb{E}_{A_t}(A_{t+i}) \{\int_0^{\omega_{t+1}^n} \mathbb{E}_{\phi_t}[\prod_{h=1}^{i} f_{\phi_{t+h}}(\omega)] \lambda(\omega)^{i-1} k_{t+1}(\omega_{t+1}^n) d\omega\}^{\alpha},$$

where $\int_0^{\omega_{t+1}^n} q_t(\omega) d\omega \int_0^{\omega_{t+1}^n} k_{t+1}(\omega_{t+1}^n) d\omega = a_{t+1}^{(\cdot,n)}$.

We extend this efficient investment interval to a loan with liquidation plan $a_{t+1}^{(j,n)}$. We define the efficient interval of capital quality, given the states A_t, ϕ_t, and a unit amount of loan $(a_{t+1}^{(j,n)} = 1)$ with loan maturity n, and liquidation plan j. If we define

$\omega_{t+1}(j,n)^* = [0, \omega_{t+1}^{(j,n)*}]$, then $\omega_{t+1}^{(j,n)*}$

$$= \arg \max_{\omega_{t+1}^n \in \omega_{t+1}(n)} [\sum_{i=1}^{j-1} \mathbb{E}_{A_t}(A_{t+i}) \{ \int_0^{\omega_{t+1}^n} \mathbb{E}_{\phi_t} [\prod_{h=1}^{i} f_{\phi_{t+h}}(\omega)] \lambda(\omega)^{i-1} k_{t+1}(\omega_{t+1}^n) d\omega \}^\alpha$$

$$+ \mathbb{P}(\Psi_{t+1}^{(j,n)}) \int_0^{\omega_{t+1}^n} q_t(\omega) d\omega \int_0^{\omega_{t+1}^n} \mathbb{E}_{\phi_t} [\prod_{i=1}^{j} f_{\phi_{t+i}}(\omega)] \lambda(\omega)^{j-1} k_{t+1}(\omega_{t+1}^n) d\omega$$

$$+ (1 - \mathbb{P}(\Psi_{t+1}^{(j,n)})) \sum_{i=j}^{n} \mathbb{E}_{A_t}(A_{t+i}) \{ \int_0^{\omega_{t+1}^n} \mathbb{E}_{\phi_t} [\prod_{h=1}^{i} f_{\phi_{t+h}}(\omega)] \lambda(\omega)^{i-1} k_{t+1}(\omega_{t+1}^n) d\omega \}^\alpha],$$

where $\int_0^{\omega_{t+1}^n} q_t(\omega) d\omega \int_0^{\omega_{t+1}^n} k_{t+1}(\omega_{t+1}^n) d\omega = 1$.

Since the interval $\omega_{t+1}(j,n)^*$ has nothing to do with the banker's discount rate, the optimal interval of the banker can be different form this interval $\omega_{t+1}(j,n)^*$. However, the interval is decided by the production level efficiency. We assume that the real investment takes place only in the set of efficient intervals $\{\omega_{t+1}(j,n)^*\}$, and so does the bank loans. And noticed that $\omega_{t+1}(j,n)^*$ depends only on (A_t, ϕ_t).

g. Deposit Contract and Incentive Constraint

Households makes deposits d_{t+1} with gross interest rate $r_{t+1}^d > 1$ at time t, only when the bank is believed to be solvent at time $t+1$. Therefore, we impose a incentive constraint such that (1) if the bank (we assume the bank is risk-averse expected utility maximizer.) solves her optimization problem with the following liquidity condition, (2) the bank can take deposits from households. The condition is that

$$for\ all\ time\ and\ states,$$

$$deposit\ repayment < \ revenues\ from\ assets\ + new\ deposit\ taking.$$

h. Producers' Problems

According to the Diamond and Rajan (2001), producers have their own technology-specific skills, and they can enjoy rents from those technologies if any. They have no endowed capital, and no storage technology. Suppose that the capital goods producer transforms the consumption goods to capital goods with the capital quality dependent costs $Q_t^{\omega_0} = \int_0^{\omega_0} q_t(\omega)d\omega, \omega_0 \in \Omega$. There will be no profits in the capital goods production. We assume that the capital goods producer can access unlimitedly to intra-period bank loan with gross interest rate $'1'$, and there is no problem for the banker to pay back the loan. A unit mass of final goods producers draw $z \in [0, 1]$ at every period, and the skills can handle the capital goods with quality $[0, z]$ are endowed to the producer z. Since they take part in the production process only with bank loan, they accept inelastically the bank's lending offer. Therefore there is nothing to optimize in producers' problems.

i. Households Problems

A unit mass of households are identified with a number $h \in [\underline{\beta}, \overline{\beta}]$. The household h gets time preference $F(h)$ at each period as following,

$$F_t(h) = \int_{\underline{\beta}}^{h} f_{g_t}(\beta)d\beta$$

Each household maximizes the expected utilities over the consumption streams,

$$\mathbb{E}[\sum_{t=0}^{\infty} \beta_t \log(c_t)]$$

Household's problem is,

$$V_t(d_t) = \max_{d_{t+1}} \log(c_t(s_t)) + \beta_t \mathbb{E}[V_{t+1}(d_{t+1}(s_{t+1}))]$$

$$s.t. {}^{\forall} t, e_t(s_t) + r_t^d(s_t)d_t(s_t) \geq c_t(s_t) + d_{t+1}(s_{t+1}),$$

$$positivities, e_0 \ given, \ and \ take \ r_t^d \ as \ given,$$

where $s_t \in S \equiv \mathcal{A} \times \Phi \times \mathcal{G}$ is the aggregate state. The total labor income $(1-\alpha)Y_t(s_t)$ is equally distributed to each personal $e_t(s_t)$, so that $(1-\alpha)Y_t(s_t) = \int_h e_t(s_t)dh$.

j. Bank Problem

For the incentive constraint to work, we assume that the banker is a expected utility maximizer, and the form is a log function. Then by the strict positivity condition of each period consumption, the incentive constraint is automatically met.

$$\mathbb{E}[\sum_{t=0}^{\infty} \beta^b \log(c_t)]$$

where β^b is the time preference of the banker. The banker sets the deposit interest rate r^d competitively, and makes loans with $'n'$ number of different maturities to the final producers. The banker should decide whether to liquidate or not against the existing loans, if the borrower requests renegotiation. The banker's revenue from loan contract will be $\min\{P_{t+j}, r_{t+j}\}$ if the contract not liquidated. If she does, she can get m_{t+j}. Therefore the banker problem contains lots of choice variables, and it extends to the $'n'$ period problem.

Before the exact and complicate algebra comes, we present the banker's problem briefly,

$$V_t(States_t) = \max_{l_t}\{\max_{a_{t+1}, r^d_{t+1}} \log(c_t(s_t)) + \beta^b \mathbb{E} V_{t+1}(States_{t+1})\}$$

$$s.t.^{\forall} t, c_t(s_t) = revenues_t^{(l_t^1)}(s_t) + revenues_t^{(l_t^0)}(s_t) - new\_loan_{t+1}(s_{t+1})$$

$$+D_{t+1}(s_{t+1}) - r_t^d D_t(s_t),$$

$$^{\forall}t, ^{\forall} s_t, c_t(s_t) > 0,$$

$$\{\omega_{t+1}(j,n) : \int_0^{\omega_{t+1}^{(j,n)}} q_t(\omega) d\omega \int_0^{\omega_{t+1}^{(j,n)}} k_{t+1}(\omega_{t+1}^{(j,n)}) d\omega = a_{t+1}^{(j,n)}\} \subset \{\omega_{t+1}(j,n)^*\},$$

$$r_{t+1}^d > 1 \ and\ take\ D_{t+1}\ as\ given$$

where, $States_t$ contains the past loan portfolios L_t, deposit D_t and liquidation histories H_{t-1}, and formally $(L_t^{H_{t-1}}, D_t)$, $L_t^{H_{t-1}}$ denotes the portfolios with liquidation histories. $Revenues_t^{(l_t^1)}$ is total revenues from the past portfolios which is not liquidated at time t, formally, $1_n'(R_t(s_t) \circ H_{t-1} \circ l_t)1_n$, the matrices will be explained soon, $revenues_t^{(l_t^0)}$ total revenues from liquidated portfolio, formally $1_n'(M_t(s_t) \circ (1-l_t) \circ T)1_n$, and new_loan_{t+1} formally $1_n' a_{t+1}(s_{t+1})$, where $\circ$ denotes elementwise multiplication in vector space. And $^{\forall} s_{t+1}, D_{t+1}(s_{t+1}) = \int_\beta f_{g_{t+1}}(\beta) d_{t+1}(s_{t+1}) d\beta$. We impose the efficient investment interval condition as a constraint.

Notations, first, should be considered,

Notation II.3. $1_n = (1, 1, \cdots, 1)'$ denote one vector size n, and assume that $n \in \mathbb{N}$ is sufficiently large.

T is lower triangular matrix which has $'1'$ for each nonzero element.

$a_{t+1}^{(n)} \in \mathbb{R}_+$ denotes the loan amounts with maturity $'n'$.

$a_{t+1} = (a_{t+1}^{(1)}, \cdots, a_{t+1}^{(n)})' \in \mathbb{R}_+^n$ denotes a loan vector.

$L_t = (a_t, a_{t-1}, \cdots, a_{t-n+1}) \circ T \in \mathbb{R}_+^n \times \mathbb{R}_+^n$ denotes a loan matrix which was made from time $t - n$.

$$ex)\ L_t = \begin{pmatrix} a_t^{(1)} & 0 & 0 & \cdot & 0 \\ a_t^{(2)} & a_{t-1}^{(2)} & 0 & \cdot & 0 \\ a_t^{(3)} & a_{t-1}^{(3)} & a_{t-2}^{(3)} & \cdot & 0 \\ . & . & . & . & . \\ a_t^{(n)} & a_{t-1}^{(n)} & a_{t-2}^{(n)} & \cdot & a_{t-n+1}^{(n)} \end{pmatrix}$$

$R_{t+1}^{(j,n)} = \min\{P_{t+1}^{(j,n)}, r_{t+1}^{(j,n)}\} \in \mathbb{R}_+$ denotes the actual loan repayment of loan a_{t+1}^n at time $t+j$.

$$R_t = \begin{pmatrix} R_t^{(1,1)} & 0 & 0 & \cdot & 0 \\ R_t^{(1,2)} & R_{t-1}^{(2,2)} & 0 & \cdot & 0 \\ R_t^{(1,3)} & R_{t-1}^{(2,3)} & R_{t-2}^{(3,3)} & \cdot & 0 \\ . & . & . & . & . \\ R_t^{(1,n)} & R_{t-1}^{(2,n)} & R_{t-2}^{(3,n)} & \cdot & R_{t-n+1}^{(n,n)} \end{pmatrix}$$ denotes a time t actual loan repayment matrix.

$$M_t = \begin{pmatrix} m_t^{(1,1)} & 0 & 0 & \cdot & 0 \\ m_t^{(1,2)} & m_{t-1}^{(2,2)} & 0 & \cdot & 0 \\ m_t^{(1,3)} & m_{t-1}^{(2,3)} & m_{t-2}^{(3,3)} & \cdot & 0 \\ . & . & . & . & . \\ m_t^{(1,n)} & m_{t-1}^{(2,n)} & m_{t-2}^{(3,n)} & \cdot & m_{t-n+1}^{(n,n)} \end{pmatrix}$$ denotes a time t loan liquidation value matrix.

$l_{t-k+1,t}^{(\cdot,n)} \in \{0,1\}$ denotes a time t liquidation over the past loan $a_{t-k+1}^{(n)}, k \leq n$, where $'1'$ if it is not liquidated, $'0'$ if liquidated.

$H_{t-1} \in \{0,1\}^n \times \{0,1\}^n$ denotes accumulated history matrix of liquidations.

$l_t \in \{0,1\}^n \times \{0,1\}^n$ denotes a time t liquidation matrix over the portfolio $L_t^{H_{t-1}}$ at time t, where $L_t^{H_{t-1}}$ means the portfolio matrix of not-liquidated upto t.

Now we present a formal banker problem,

$$V_t(L_t^{H_{t-1}}, D_t) = \max_{l_t}\{ \max_{a_{t+1}, r_{t+1}^d} \log(c_t(s_t)) + \beta^b \mathbb{E} V_{t+1}(L_{t+1}^{H_t}, D_{t+1})\}$$

$$s.t.^{\forall} t, c_t(s_t) = 1_n'(R_t(s_t) \circ H_{t-1} \circ l_t)1_n + 1_n'(M_t(s_t) \circ (1 - l_t) \circ T)1_n - 1_n' a_{t+1}(s_{t+1})$$

$$+ D_{t+1}(s_{t+1}) - r_t^d D_t(s_t),$$

$$^{\forall} t, ^{\forall} s_t, c_t(s_t) > 0,$$

$$\{\omega_{t+1}(j,n) : \int_0^{\omega_{t+1}^{(j,n)}} q_t(\omega) d\omega \int_0^{\omega_{t+1}^{(j,n)}} k_{t+1}(\omega_{t+1}^{(j,n)}) d\omega = a_{t+1}^{(j,n)}\} \subset \{\omega_{t+1}(j,n)^*\},$$

$$r_{t+1}^d > 1 \textit{ and take } D_{t+1} \textit{ as given}$$

k. Time of Events

At the beginning of each period, the state s_t revealed. Then renegotiation and liquidation take place. After productions are conducted, incomes distributed and agents make deliveries according to their contracts. Financial markets open and then agents consume.

2. Market Clearing Conditions and Equilibrium

There are two financial markets and two goods markets. Loan market always automatically clears, and also the deposit market clears by the model setting. The banker take $^{\forall} s_{t+1}, D_{t+1}(s_{t+1}) = \int_\beta f_{g_{t+1}}(\beta) d_{t+1}(s_{t+1}) d\beta$ as given. By the Wallas's Law, therefore, we need to clear only one market, the capital goods market. And the condition is

$$^{\forall} s_{t+1}, 1_n' a_{t+1}(s_{t+1}) = \sum_{\omega_{t+1}^{(j,n)*}} \int_0^{\omega_{t+1}^{(j,n)*}} q_t(\omega) d\omega \int_0^{\omega_{t+1}^{(j,n)*}} k_{t+1}(\omega_{t+1}^{(j,n)*}) d\omega.$$

We need a consistency condition which stems from the difference between the capital in actual production process and that of banker's total loans due to the results of liquidations. So, we states roughly that total economy's capital should equal to the sum of banker's total loan and liquidated loan, and formally

$$
\begin{aligned}
{}^{\forall}s_t, K_t(s_t) = &\sum_{\substack{i=1\\h\geq i}}^{n} \sum_{\omega_{t-i+1}^{(j,h)*}} \int_0^{\omega_{t-i+1}^{(j,h)*}} [\prod_{u=1}^{i} f_{\phi_{t-i+u}}(\omega)] \lambda(\omega)^{i-1} k_{t-i+1}^{\{l_t^{(j,h)}=1\}}(\omega_{t-i+1}^{(j,h)*}) d\omega \\
&+ \sum_{\substack{i=1\\h\geq i}}^{n} \sum_{\omega_{t-i+1}^{(j,h)*}} \int_0^{\omega_{t-i+1}^{(j,h)*}} [\prod_{u=1}^{i} f_{\phi_{t-i+u}}(\omega)] \lambda(\omega)^{i-1} k_{t-i+1}^{\{l_t^{(j,h)}=0\}}(\omega_{t-i+1}^{(j,h)*}) d\omega,
\end{aligned}
$$

where $K_t(s_t)$ denotes economy's total capital goods in production process at time t, and the first summation $\sum_{\substack{i=1\\h\geq i}}^{n}$ means the sum all capitals which has maturity h traded i periods before.

Definition II.4. (Recursive Competitive Equilibrium) A recursive competitive equilibrium is a set of gross deposit interest rate $\{r^d(S)\}$, a set of policy functions for households $\{c^H(S), d(S)\}$, a set of policy functions for the banker $\{c^B(S), a(S), l(S), D(S)\}$ and the probability measure Γ satisfies the followings,(1) households' policy functions are solutions to their problems taking $r^d(S), \Gamma$ given, (2) banker's policy functions are solutions to her problems with the incentive constraint, taking Γ given, (3) capital goods market clears. And a set of total capitals $\{K(S)\}$ with liquidation policy functions $\{l(S)\}$ satisfies the consistency conditions.

C. Characterization

1. Policy Functions

a. Aggregate Deposit of Households and The Banker's Choice of r^d

Since the banker can set the deposit interest rate r^d, and since the households income streams directly depend on the banker's behavior, the banker can incorporate the households problems into her problem. From the households optimizations, given states and r^d, the banker's total deposit taking will be the following,

$$D_{t+1}(r_{t+1}^d; s_{t+1}, \Delta_{t+1}) \equiv \int\int f_{g_t}(\beta) d_{t+1}^*(\beta, r_{t+1}^d; s_{t+1}, \Delta_{t+1}) d\beta dh$$

where $d_{t+1}^*(\beta, r_{t+1}^d; s_{t+1}, \Delta_{t+1})$ is the optimal choice of household at time t, and Δ_{t+1} is the banker's loan portfolio states, which will be explained soon. The household's optimal deposit $d_{t+1}^*(\beta, r_{t+1}^d; s_{t+1}, \Delta_{t+1})$ comes from the first order condition of the households problem.

$$1 = \mathbb{E}_{t,t+1}[\frac{\beta c_t(s_t)}{c_{t+1}(s_{t+1})} r_{t+1}^d(s_{t+1})],$$

where $c_t(s_t) = (1-\alpha)A_t(\Delta_t^*)^\alpha + r_t^d d_t(\beta, r_t^d; s_t, \Delta_t) - d_{t+1}(\beta, r_{t+1}^d; s_{t+1}, \Delta_{t+1})$.

Then the banker sets r^d the aggregate deposit $D_{t+1}(r_{t+1}^d; s_{t+1}, \Delta_{t+1})$ satisfies her FOC,

$$1 = \mathbb{E}_{t,t+1}[\frac{\beta^b c_t^b(s_t)}{c_{t+1}^b(s_{t+1})} r_{t+1}^d(s_{t+1})],$$

where $c_t^b(s_t)$ is the banker's consumption.

b. Simple Function Representation of Bank Optimal Portfolio

We denote

$W_{t+1}^{(j,n)}(L_t^{H_{t-1}}, D_t; s_t)$ the discounted sum of revenues from the loan $a_{t+1}^{(j,n)}(L_t^{H_{t-1}}, D_t; s_t)$ with maturity n and liquidation plan j, then

$$\begin{aligned} W_{t+1}^{(j,n)}(L_t^{H_{t-1}}, D_t; s_t) = & \sum_{i=1}^{j-1} \mathbb{E}_{t,t+i} \frac{(\beta^b)^i u'(c_{t+i}(s_{t+i}))}{u'(c_t(s_t))} R_{t+1}^{(i,n)} \\ & + \mathbb{P}(\Psi_{t+1}^{(j,n)}) \mathbb{E}_{t,t+j} \frac{(\beta^b)^j u'(c_{t+j}(s_{t+j}))}{u'(c_t(s_t))} m_{t+1}^{(j,n)} \\ & + (1 - \mathbb{P}(\Psi_{t+1}^{(j,n)})) \sum_{i=j}^{n} \mathbb{E}_{t,t+i} \frac{(\beta^b)^i u'(c_{t+i}(s_{t+i}))}{u'(c_t(s_t))} R_{t+1}^{(i,n)}, \end{aligned}$$

where $\Psi_{t+1}^{(j,n)} \equiv \{\psi_{t+1}^{(j,n)} \in \mathcal{U}^{(j,\cdot)} : r_{t+j} < P_{t+j}$, and $r_{t+j} < m_{t+j}$, given $a_{t+1}^{(j,n)}, A_t, \phi_t\}$ as defined above. And for the loan of no liquidation plan $a_{t+1}^{(0,n)}(L_t^{H_{t-1}}, D_t; s_t)$,

$$W_{t+1}^{(0,n)}(L_t^{H_{t-1}}, D_t; s_t) = \sum_{i=1}^{n} \mathbb{E}_{t,t+i} \frac{(\beta^b)^i u'(c_{t+i}(s_{t+i}))}{u'(c_t(s_t))} R_{t+1}^{(i,n)}.$$

By the *assumption II.2.(b)*, we know that the discounted sum of revenues of long term loan is always bigger than that of short term loan, so we have

$$^{\forall}n < m, W_{t+1}^{(0,n)}(L_t^{H_{t-1}}, D_t; s_t) < W_{t+1}^{(0,m)}(L_t^{H_{t-1}}, D_t; s_t).$$

We assume that the discounted sum of future revenues is always bigger than the current value of capital. This assumption is quite natural. If it is not, we don't need to have production process. By this assumption with the *assumption 2.(a)*, the discounted sum of early liquidation plan is always smaller than that of late liquidation plan, so we have

$$^{\forall}i < j \le n, W_{t+1}^{(i,n)}(L_t^{H_{t-1}}, D_t; s_t) < W_{t+1}^{(j,n)}(L_t^{H_{t-1}}, D_t; s_t) < W_{t+1}^{(0,n)}(L_t^{H_{t-1}}, D_t; s_t).$$

Proposition II.5. *(a)* $^{\forall}n < m, W_{t+1}^{(0,n)}(L_t^{H_{t-1}}, D_t; s_t) < W_{t+1}^{(0,m)}(L_t^{H_{t-1}}, D_t; s_t)$. *(b)* $^{\forall}i < j \le n, W_{t+1}^{(i,n)}(L_t^{H_{t-1}}, D_t; s_t) < W_{t+1}^{(j,n)}(L_t^{H_{t-1}}, D_t; s_t) < W_{t+1}^{(0,n)}(L_t^{H_{t-1}}, D_t; s_t)$.

We remember that $\omega_{t+1}^{(j,n)*}(A_t, \phi_t)$ is fixed on the state space of (A_t, ϕ_t). Then, the FOC of the banker problem for $a_{t+1}^{(j,n)}(L_t^{H_{t-1}}, D_t; s_t)$ is

$$1 = W_{t+1}^{(j,n)}(L_t^{H_{t-1}}, D_t; s_t),\ \omega_{t+1}^{(j,n)*}(A_t, \phi_t) \text{ given.}$$

Therefore, with this FOCs and the *proposition 5.(a)* show that there can be only one optimal maturity if we do not allow liquidation, and the *(b)* tells us that there can be multiple optimal maturities with liquidation plans, and if a loan with no liquidation plan is optimal, then we can't find optimal among the shorter maturity loans.

We suppose a set of optimal loan portfolios with the fixed efficient investment interval of capital quality, $\{(a_{t+1}^{(j,n)*}(L_t^{H_{t-1}}, D_t; s_t), \omega_{t+1}^{(j,n)*}(A_t, \phi_t))\}$. The each element of the optimal portfolio set can be represented on the plane of $R_+ \times \Omega$, since

$$\begin{aligned} a_{t+1}^{(j,n)*}(L_t^{H_{t-1}}, D_t; s_t) &= \int_0^{\omega_{t+1}^{(j,n)*}} q_t(\omega) d\omega \int_\omega X_{[0,\omega_{t+1}^{(j,n)*}]} k_{t+1}^{(j,n)*} d\omega \\ &= Q_t(\omega_{t+1}^{(j,n)*}) \int_\omega X_{[0,\omega_{t+1}^{(j,n)*}]} k_{t+1}^{(j,n)*} d\omega, \end{aligned}$$

where $\int_0^{\omega_{t+1}^{(j,n)*}} q_{t+1}(\omega) d\omega = Q_{t+1}(\omega_{t+1}^{(j,n)*}) \in \mathbb{R}_+$, since $Q_{t+1}(\omega_{t+1}^{(j,n)*})$ depend only on the interval $[0, \omega_{t+1}^{(j,n)*}]$, and $X_{[0,\omega_{t+1}^{(j,n)*}]}$ is an indicator function.

Now, we have a simple function representation of the optimal portfolio set,

$$\sum_{\omega_{t+1}^{(j,n)*}} a_{t+1}^{(j,n)*} = \sum_{\omega_{t+1}^{(j,n)*}} Q_{t+1}(\omega_{t+1}^{(j,n)*}) k_{t+1}^{(j,n)*} \int_\omega X_{[0,\omega_{t+1}^{(j,n)*}]} d\omega \equiv \delta_{t+1}^*(L_t^{H_{t-1}}, D_t, \omega_{t+1}^{(j,n)*}; s_t).$$

Proposition II.6. *The banker's optimal loan portfolio has a simple function representation.*

With notational abuse, we denote the physical capital using the simple function notation as follows,

$$\delta_{t+1}/Q_{t+1} \equiv \sum_{\omega_{t+1}^{(j,n)*}} k_{t+1}^{(j,n)*} \int_\omega X_{[0,\omega_{t+1}^{(j,n)*}]} d\omega.$$

2. Two Period Representation of $'n'$ Period Problem

a. Simple Function Representation of Portfolio

We denote Δ_t the state of loan portfolio at time t, then

$$\Delta_t =$$

$$\sum_{\substack{i=1\\h\geq i}}^{n}\sum_{\omega_{t-i+1}^{(j,h)*}}[\prod_{u=0}^{i-2} l_{t-i+1,t-i+1+u}^{(j,h)}]Q_{t-i+1}(\omega_{t-i+1}^{(j,h)*})k_{t-i+1}^{(j,h)*}\int_{\omega} X_{[0,\omega_{t-i+1}^{(j,h)*}]}[\prod_{u=1}^{i} f_{\phi_{t-i+u}}(\omega)]\lambda(\omega)^{i-1}d\omega$$

where $\prod_{u=0}^{i-2} l_{t-i+1,t-i+1+u}^{(j,h)} = 1$, if $a_{t-i+1}^{(j,h)*}$ is not liquidated from time $t-i+1$ to time $t-1$, $\prod_{u=0}^{i-2} l_{t-i+1,t-i+1+u}^{(j,h)} = 0$, if $a_i^{(j,h)*}$ is liquidated before time $t-1$, therefore Δ_t is the sum of all loans which is not liquidated and maturity is not passed $h \geq i$.

Then we can denote the state of physical capital induced by the state of loan portfolio Δ_t/Q_t,

$$\Delta_t/Q_t = \sum_{\substack{i=1\\h\geq i}}^{n}\sum_{\omega_{t-i+1}^{(j,h)*}}[\prod_{u=0}^{i-1} l_{t-i+1,t-i+1+u}^{(j,h)}]k_{t-i+1}^{(j,h)*}\int_{\omega} X_{[0,\omega_{t-i+1}^{(j,h)*}]}[\prod_{u=1}^{i} f_{\phi_{t-i+u}}(\omega)]\lambda(\omega)^{i-1}d\omega.$$

With notational abuse, we denote $\Delta_t = l_t^1\Delta_t + l_t^0\Delta_t$, where $l_t^1\Delta_t$ is the not liquidated time t portfolio state, $l_t^0\Delta_t$ is the liquidated time t portfolio state.

$$\begin{aligned} l_t^1\Delta_t = &\sum_{\substack{i=1\\h\geq i}}^{n}\sum_{\omega_{t-i+1}^{(j,h)*}}[\prod_{u=0}^{i-1} l_{t-i+1,t-i+1+u}^{(j,h)}]Q_{t-i+1}(\omega_{t-i+1}^{(j,h)*})k_{t-i+1}^{(j,h)*} \\ &\times \int_{\omega} X_{[0,\omega_{t-i+1}^{(j,h)*}]}[\prod_{u=1}^{i} f_{\phi_{t-i+u}}(\omega)]\lambda(\omega)^{i-1}d\omega. \end{aligned}$$

$$l_t^0\Delta_t = \sum_{\substack{i=1\\h\geq i}}^{n} \sum_{\omega_{t-i+1}^{(j,h)*}} [1 - \prod_{u=0}^{i-1} l_{t-i+1,t-i+1+u}^{(j,h)}] Q_{t-i+1}(\omega_{t-i+1}^{(j,h)*}) k_{t-i+1}^{(j,h)*}$$
$$\times \int_{\omega} X_{[0,\omega_{t-i+1}^{(j,h)*}]}[\prod_{u=1}^{i} f_{\phi_{t-i+u}}(\omega)]\lambda(\omega)^{i-1}d\omega.$$

Then $\Delta_{t+1} = l_t^1\Delta_t + \delta_{t+1}^*$.

And the physical capital states have,

$$l_t^1\Delta_t/Q_t = \sum_{\substack{i=1\\h\geq i}}^{n} \sum_{\omega_{t-i+1}^{(j,h)*}} [\prod_{u=0}^{i-1} l_{t-i+1,t-i+1+u}^{(j,h)}] k_{t-i+1}^{(j,h)*}$$
$$\times \int_{\omega} X_{[0,\omega_{t-i+1}^{(j,h)*}]}[\prod_{u=1}^{i} f_{\phi_{t-i+u}}(\omega)]\lambda(\omega)^{i-1}d\omega,$$

$$l_t^0\Delta_t/Q_t = \sum_{\substack{i=1\\h\geq i}}^{n} \sum_{\omega_{t-i+1}^{(j,h)*}} [1 - \prod_{u=0}^{i-1} l_{t-i+1,t-i+1+u}^{(j,h)}] k_{t-i+1}^{(j,h)*}$$
$$\times \int_{\omega} X_{[0,\omega_{t-i+1}^{(j,h)*}]}[\prod_{u=1}^{i} f_{\phi_{t-i+u}}(\omega)]\lambda(\omega)^{i-1}d\omega,$$

and define an operator $\Upsilon_{(f_\phi,\lambda)}(X)(\omega) = \int f_\phi(\omega)\lambda(\omega)X(\omega)d\omega$, and $\Upsilon_{(f_\phi)}(X)(\omega) = \int f_\phi(\omega)X(\omega)d\omega$, then the beginning of time $t+1$ physical capital state Δ_{t+1}/Q_{t+1} is

$$\Delta_{t+1}/Q_{t+1} = \Upsilon_{(f_{\phi_{t+1}},\lambda)}(l_t^1\Delta_t/Q_t) + \Upsilon_{(f_{\phi_{t+1}})}(\delta_{t+1}/Q_{t+1}).$$

b. Budget Set

We can get the banker's budget set as of the simple functional form. The total time t revenues from existing loans will be

$$\gamma\{\alpha A_t[l_t^1\Delta_t/Q_t]^\alpha + Q_t(l_t^0\Delta_t/Q_t)\},$$

where $Q_t(l_t^0(\Delta_t)/Q_t)$ is the total revenues from liquidations with notational abuse. Then we have the time t budgets given the liquidation matrix l_t as follows,

$$c_t(s_t) = \gamma\alpha A_t[\Upsilon_{(f_{\phi_t})}(l_t^1\Delta_t/Q_t)]^\alpha + \gamma Q_t(l_t^0\Delta_t/Q_t) - \delta_{t+1}(s_{t+1}) + D_{t+1}(s_{t+1}) - r_t^d D_t(s_t).$$

c. The Banker's Optimal Portfolio Problem

With the above simple function representations of optimal loan policy, and loan portfolio states, we can change the banker's $'n'$ periods problem to $'2'$ periods problem. Due to this characterization, we can find the equilibrium sets of policies, values and prices by computations.

$V_t(\Delta_t/Q_t, D_t, s_t) \max_{l_t}\{\max_{r^d_{t+1},\delta_{t+1}} \log(c_t(s_t)) + \beta^b \mathbb{E}V_{t+1}(\Delta_{t+1}/Q_{t+1}, D_{t+1}, s_{t+1})\}$

$s.t.{}^\forall t, c_t(s_t) = \gamma\alpha A_t[\Upsilon_{(f_{\phi_t})}(l_t^1\Delta_t/Q_t)]^\alpha + \gamma Q_t(l_t^0\Delta_t/Q_t) - \delta_{t+1}(s_{t+1}) + D_{t+1}(s_{t+1}) - r_t^d D_t(s_t),$

${}^\forall t, {}^\forall s_t, c_t(s_t) > 0,\ and\ r_t^d(s_t) > 1,$

$\omega_{t+1}^{(j,n)*}(s_t), Q_{t+1}(\omega_{t+1}^{(j,n)*}),\ D_{t+1}(r^d_{t+1}; s_{t+1}, \Delta_{t+1}) \equiv \int f_{g_t}(\beta) d^*_{t+1}(\beta, r^d_{t+1}; s_{t+1}, \Delta_{t+1}) d\beta$ given.

Since the simple function has the following rectangular shape, the loan portfolio choice problem changes to a problem choosing the height $k_{t+1}^{(j,n)*}$ of the given width of the rectangular.

$V_t(\Delta_t, D_t, s_t) = \max_{l_t}\{\max_{r^d_{t+1},\{k^{(j,n)*}_{t+1}\}} \log(c_t(s_t)) + \beta^b \mathbb{E}V_{t+1}(\Delta_{t+1}, D_{t+1}, s_{t+1})\}$

$s.t.{}^\forall t, c_t(s_t) = \gamma\alpha A_t[\Upsilon_{(f_{\phi_t})}(l_t^1(\Delta_t)/Q_t)]^\alpha + \gamma Q_t(l_t^0(\Delta_t)/Q_t)$
$- \sum_{\omega^{(j,n)*}_{t+1}}\{Q_{t+1}(\omega^{(j,n)*}_{t+1}) k^{(j,n)*}_{t+1} \int_\omega X_{[0,\omega^{(j,n)*}_{t+1}]} d\omega\} + D_{t+1}(s_{t+1}) - r_t^d D_t(s_t),$

${}^\forall t, {}^\forall s_t, c_t(s_t) > 0,\ and\ r_t^d(s_t) > 1,$

$\omega_{t+1}^{(j,n)*}(s_t),\ Q_{t+1}(\omega_{t+1}^{(j,n)*})$ given.

Take $D_{t+1}(r^d_{t+1}; s_{t+1}, \Delta_{t+1}) \equiv \int\int f_{g_t}(\beta) d^*_{t+1}(\beta, r^d_{t+1}; s_{t+1}, \Delta_{t+1}) d\beta dh$ as given.

D. Quantitative Analysis

In this section, we present three numerical experiments designed to illustrate how the bank achieves the optimal loan portfolio, total deposit taking and the optimal liquidation plan. The first experiment will show the bank optimal behavior in normal economic situation, and then we change some parameter values and transition matrices for the model to resemble an economy in recession (we call the economy "productivity shocked economy"). Moreover, we postulate a "liquidity shocked economy," where households consume more, and save less due to the pessimistic expectation for the future.

1. Computation

Using simple function characterization of our problem, we construct a loan portfolio state space. It has n dimensional size. Our optimality condition gives us a method of reduction technique. First, the simple functional forms will have a limited type. For instance, if $n = 3$, the total types of simple function only 10. And second, they are ordered by optimality.[1] Therefore, the space is sectorized according to the order. For a multi-assets sector, there will be a non-visited area. For example, the non visited area violates the proposition 5. The number of grids reduces to 1/4 for the section of

[1]The 10 types of simple functions are ordered by the optimality, and the order is,
1. $a^{(3)}$
2. $a^{(2)}, a^{(2,3)}$
3. $a^{(2)}, a^{(1,3)}$
4. $a^{(2)}$
5. $a^{(1)}, a^{(1,2)}, a^{(2,3)}$
6. $a^{(1)}, a^{(1,2)}, a^{(1,3)}$
7. $a^{(1)}, a^{(2,3)}$
8. $a^{(1)}, a^{(1,3)}$
9. $a^{(1)}, a^{(1,2)}$
10. $a^{(1)}$.

two assets, and $1/48$ for that of three assets. Figure 1 shows the initial loan portfolio of a bank.

As a result, the efficient intervals are figured by the production side technologies, which is largely depend on depreciation, and capital goods cost factors. We set up these functions are satisfied with the technology arbitrage conditions. Next, households problems are incorporated in bank's problem, and also producers have nothing to choose.

2. Parameters

We have three types of parameters, which are macro economic parameters (production technology A, α, time preference for household and bank β, β^b, and the banker's loan collection skill γ), capital quality parameters (remaining portion after depreciation $\lambda(\omega)$, capital goods price $q(\omega)$, and maturity factor η), and distributional parameters (for household distribution over time preference $F_g(h)$ and capital quality shock $f_\phi(\omega)$). The efficient intervals of capital quality is very sensitive to the three functions $f_\phi(\omega)$, $\lambda(\omega)$, $q(\omega)$. We set up those functions to guarantee $'3'$ loans in the model economy. Table I displays specific values and functions which we adopt for calibration.

3. Standardized Loan Contracts

From the specific parameter values and function forms, we can standardize loan contracts. The intervals $\omega_t(n) = \{\omega : N(\omega_0, \eta, \phi_t) = n\}$ of capital qualities and the efficient investment capital qualities $\omega_t^{(\cdot,n)*}$ for different loan maturities are shown in table II. For example, producer should hire the interval of capital quality from $[0, 0.9077]$ to $[0, 1]$, for the maturity $'3'$ period investment, and the efficient investment can be done when the producer hires $[0, 0.9077]$. For the $'2'$ period loan, the efficient level of investment is $[0, 0.5336]$, and $[0, 0.0967]$ for $'1'$ period investment.

Table III shows the stream of revenues for the different maturity loans, and table IV exhibits liquidation values. The banker's revenues from $'3'$ period loan are 0.7150, 0.6286, 0.5661 each period for the unit amount of loan, when the states is "good" and "low." Therefore the total revenue of $'3'$ period loan is 1.9097, which is the highest. When the banker lends one unit amount for $'2'$ period, she expects total revenue of 1.586. For the $'1'$ period loan, 1.2571 of revenue is expected. The shorter loan has more liquidity, but not profitable. The longer loan promises bigger revenue, but less liquid. The liquidation values are less than the investment amount, therefore liquidation means a loss.

4. Simulation Strategy

We will examine the banker's optimal loan portfolio, deposit taking and liquidation plan given the economy's states. First, we find out the optimalities when the states are normal. Then, we makes the economy in the middle of recession, where the productivity is low, and it is persistent. Lastly, a deep recession will be simulated, where deposit reduces and it is also persistent, and the maximum of deposit interest rate rises from 1.5 to 2.0.

For the three different shocks, we set transition matrices for the "normal economy", "productivity shocked economy", and "liquidity shocked economy," by table V, table VI, and table VII, respectively.

5. Results

The summary results of (1) normal economy, (2) productivity shocked economy, and (3) liquidity shocked economy are shown by VIII, where $(\mathcal{A} \times \Phi \times \mathcal{G})$ is in (good,low,normal) and in (bad,high,shock) states, respectively. Figures from 2 to 9 show the results of normal economy, figures from 10 to 17 show the results of productivity shocked economy, and figures from 18 to 25 show the results of liquidity shocked economy.

When a "normal" economy is in (good,low,normal) states, the banker takes 2.1576 amount as deposits from households, and lends 7.1108 amount to producers, and gets 9.6167 amount as revenues from the loans. While a "liquidity shocked" economy is in (bad,high,shock) states, the banker takes 1.6810 amount as deposits from households, and lends 2.7105 amount to producers, and gets 5.5245 amount as revenues from the loans.

a. Loan Portfolio

The normal economy has whole period loan portfolio, whereas the productivity shocked economy in (bad,high,shock) states has one $a^{(1)}$ and three $a^{(3)}$ period loans, and the liquidity shocked economy in (bad,high,shock) states has only the shortest term loan $a^{(1)}$. The normal economy in (good,low,normal) states has 72.6% of loans as the shortest maturity, 13.3% as two period, and 14.1% as the longest maturity. If the states changes to (bad,high,shock) states, the proportion of the shortest term loan is increased to 75.2%, and that of the mid term is decreased to 10.7%. Whereas the longest term shows no changes in the proportion.

When economies are in (good,low,normal) states, if economy is changed from productivity shocked to liquidity shocked, then the proportion of the longest term loan reduced from 14.1% to 12.1%, and the mid term loan increased from 10.7% to 12.7%.

b. Aggregate Deposit and Interest Rate

Productivity shock cannot affect total deposit, however liquidity shock reduces total deposit 17.5% in (good,low,normal) states, and 21.0% in (bad,high,shock) states. When the states of economy changes from (good,low,normal) to (bad,high,shock), total deposit of "normal" and "productivity shocked" economies decreases 1.4%, while that of "liquidity shocked" economy decreases 5.6%. The interest rate of deposit wouldn't change with the productivity shock, but the liquidity shock could affect the interest rate.

c. Total Output and Revenue of Bank

The total output and bank revenue are not different so big between (good,low,normal) and (bad,high,shock) states, when the economy is in "normal." However, it reduced by 35.8% when the economy was in "productivity shocked."

E. Conclusion

Long term asset creates more revenue, however it is riskier in a liquidity sense. Our question is: How does a liquidity constrained bank make decisions between profitability and liquidity? We present a computable DSGE model of banks optimal portfolio choices under liquidity constraints. We borrow major concepts from Diamond and Rajan (2001) : relationship lending, specialized skills in loan collection, renegotiation, and liquidation process. Our theory predicts that liquidation plays an important role in a bank's portfolio model. Even though liquidation is an off-equilibrium phenomenon, banks can have rich loan portfolios due to the possibility of liquidation. Liquidity condition is a key factor in banks portfolio.

Our model provides a good understanding of the possibility of loan liquidations.

This possibility ensures a bank have a portfolio of loans with different maturities, even if they do not have credit risks. We analyze the banks' loan business with different maturities. This is not easy, because of the curse of dimensions. We provide a useful method to evade this computational problem in a very limited sense.

Long term loans are profitable for the banker and provide benefits to the producers and the households, since it produces more. However, they have a limitation in liquidity. When a liquidity shock is expected, loan maturities will be shorter.

CHAPTER III

SECURITIZATION AND BANK OPTIMAL PORTFOLIO CHOICE

A. Introduction

As Rajan (2006) pointed out, banking business has changed to more risky areas, due to technical change, deregulation and institutional change. Banks improve their risk management abilities and risk taking capacities. This trend is an inevitable consequence of severe competitions. The above three factors have pushed up the banks for more profitability to survive. Banks have to involve in high profit business like securities, derivatives and exotic new financial products. Boyd and Nicolo (2005) argued that the risky business of banks is a rational choice due to the competition. Banking business is quite different from their traditional aspects. They have many securities and trading assets in their balance sheets. Without the securitization, we could not explain nowadays banking business properly. Securitization in banking business makes better off our economy in good seasons definitely. However, with the depressed securities market, banks can deepen the business cycles. Banks' debts are largely short term deposit. Many literatures analyze the securities business in banking, however, very few directly connecting the securities to the bank's short term liabilities. We provide a useful tool to analyze bank's traditional loan and deposit business connected with new securities business.

Recent sub-prime mortgage crisis is believed that banks misalignments of resources is a main cause. There are many literatures analyzing this crisis with financial frictions(Gertler, Kiyotaki, and Queralto (2010)). Our model provides why banks should involve in securities business, and how crisis evolves inherently in the bank's securities business.

Our theory predicts that the bank securities business makes a chasm between a real liquidity of economy and market liquidity. Banks can have more liquidity by selling their securitized loans, and as our previous model already pointed out, a good liquidity condition push the bank more profitable but less liquid long term loans. As a consequence, long-term loans are accumulated, and every participant happy with this securitization, simply because long term loan gives higher revenue. However, the economy has less liquidity in short term. Any market turbulence can invoke a problem in economy wide liquidity. Therefore bank regulators and macroeconomic policy agencies should make a brake to the securities business of banks. We suppose a liquidity taxation in banks' securities business as a provision for the shortage of liquidity and government direct money injections to the market as a crisis response.

B. The Model Outline

We define securitization in banking business selling loan contract with market price. Compared with liquidation, bank sell the contract at any time it is optimal, and the selling price depends on the future revenues not on the liquidation value. The selling of loan contract is on the equilibrium path, whereas the liquidation only takes place on the off equilibrium path. The optimal amount of $a_{t+1}^{(j,n)}(E_t, L_{t-1}, D_t; s_t)$ which will be securitized and sold time $t+j,\ 1 \leq j \leq n$,comes from

$$1 = \sum_{k=1}^{j-1} \mathbb{E}_{t,t+k} \frac{u'(c_{t+k}(s_{t+k}))}{u'(c_t(s_t))} R_{t+1}^{(k,n)} + \mathbb{E}_{t,t+j} \frac{u'(c_{t+j}(s_{t+j}))}{u'(c_t(s_t))} Q_{t+j}^{(j,n)}$$

where $Q_{t+j}^{(j,n)} = \psi \sum_{k=j}^{n} \mathbb{E}_{t+j,t+k} \Lambda_{t+j,t+k} R_{t+1}^{(k,n)}$ is the selling price, and $\Lambda_{t+j,t+k}$ is the buyer's stochastic discount factor, $\psi < 1$ represents some cost in securitization and also lower collection skills of buyer or less efforts of the banker on the loan collection she already sold out. Since the selling of $a_{t+1}^{(j,n)}$ is on the equilibrium path, the loan contract will be considered as a $'j'$ period asset not $'n'$ period asset. The liquidity map of the banker is good enough to lend more profitable long-term loans. Output, income, and profit increase by this securitization. As a result, good liquidity conditions of the bank will reduce the banker's attraction to taking deposits. In addition, real liquidity in economy reduces by these two directions: actual investment on long term projects and decreased creation of new liquidity. The US economy before the recent mortgage crisis showed a great boom of securitization with liquidity depletion. As expected, a crisis any time possible with a bad shock in this situation.

Our model can explain the cause of the crisis and suggest policy responses to the securitization of banking business. Suppose a shock occurs in securities market, so that some securities loose the marketability. Then the assets for securities are no longer considered as $'j'$ term assets, it should be considered to be $'n'$ period assets. This makes the liquidity condition of the bank worse all at once. The bank's default risk and the bank-run risk will be very high. The banker operates cash and short-term loan more to secure her viability. Thus output and income decrease, so deposit. These are the results of deep securitization and a sudden shock.

C. Preliminary Results

We conducted some experimental simulations using the same framework of the previous chapter. So, the result is limited, in that the households' behaviors are not fully analyzed. Our simple and primitive computation suggests that securitization in banking business makes the bank have longer term loans even if a liquidity shock is expected. Output and bank's revenue are bigger when the bank involves in securitization. The summary results are shown by IX.

D. Conclusion

Banking business is quite different from their traditional aspects. We cannot separate a securities business from nowadays banking industry. They have many securities and trading assets in their balance sheets. Without the securitization, we could not explain nowadays banking business properly. Securitization in banking business makes better off our economy in good seasons definitely. However, with the depressed securities market, banks can deepen the business cycles. Bank's debts are largely short term deposit. Our theory predicts that the bank securities business produces a chasm between a real liquidity of economy and market liquidity. Banks can have more liquidity by selling their securitized loans, and as our model already pointed out, a good liquidity condition makes the bank more profitable but less liquid long term loans. As a consequence, long term loans are accumulated and every participant is happy with this securitization, simply because a long term loan gives higher revenue. However, the economy has less liquidity in short term. Any market turbulence can invoke a problem in economy wide liquidity.

Recent mortgage crisis cause us to reconsider the securities business of a bank. Our model can explain the liquidity depletion with the progress of bank's securitization as we saw in the early 2000's. Banks securitization makes liquidity confusions. This can not be a problem in good times, however, it can be a severe problem. Even we cannot notice this real liquidity depletion when economy is in a boom. We will have a model of this process and can suggest policy responses.

CHAPTER IV

CONCLUSION

Our model provides a good understanding of the possibility of loan liquidations. This possibility ensures a bank have a portfolio of loans with different maturities, even if they do not have credit risks. We analyze the banks' loan business with different maturities. This is not easy, because of the curse of dimensions. We provide a useful method to evade this computational problem in a very limited sense.

Long term loans are profitable for the banker and provide benefits to the producers and the households, since it produces more. However, they have a limitation in liquidity. When a liquidity shock is expected, loan maturities will be shorter.

We cannot separate a securities business from nowadays banking industry. Securitization provides many benefits to the economy. However, recent mortgage crisis cause us to reconsider the securities business of a bank. Our model can explain the liquidity depletion with the progress of bank's securitization as we saw in the early 2000's. Banks securitization makes liquidity confusions. This can not be a problem in good times, however, it can be a severe problem. Even we cannot notice this real liquidity depletion when economy is in a boom. We will have a model of this process and can suggest policy responses.

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APPENDIX A

HOUSEHOLD OPTIMAL PORTFOLIO PROBLEM

- To find solution concepts of our model.

I. Environments

1. Time, $t \in \{0, 1, 2, 3\}$

2. Endowments

- $e_0 > 0, e_1 > 0, e_2 = e_3 = 0.$

3. Expenditure shock at time $t = 1$

- $h_1^S > 0$ with $\Pr(h_1^S) = p, h_1^N = 0$ with $\Pr(h_1^N) = 1 - p$

4. Two assets

- long-term assets (a^L) : one unit of a_{t+1}^L gives 3 period-coupons

$$(\delta^L R_{t+1}^L, \delta^L R_{t+2}^L, \delta^L R_{t+3}^L) = (.68, .41, .24)$$

- short-term assets (a^S) : one unit of a_{t+1}^S gives 2 period-coupons

$$(\delta^S R_{t+1}^S, \delta^S R_{t+2}^S) = (.78, .34)$$

- each asset can be liquidated before matures with it's liquidation value : if a_{t+1}^L is liquidated ($l_{t+1}^L = 1$), then the liquidation value is $\delta^L l_{t+1}(a_{t+1}^L) = .774$, and a_{t+1}^S, $\delta^S l_{t+1}(a_{t+1}^S) = .782$ for the unit amount of investment.

*** Final goods production for reference**

- High quality technology (I^H) : one unit of i_t gives 3 period-coupons

$$(R^L_{t+1}(s_{t+1}), R^L_{t+2}(s_{t+2}), R^L_{t+3}(s_{t+3})) = ((.76(G), .73(B)), (.46(G), .44(B)), (.28(G), .26(B)).$$

- Low quality technology (I^L) : one unit of i_t gives 2 period-coupons

$$(R^S_{t+1}(s_{t+1}), R^S_{t+2}(s_{t+2})) = ((.87(G), .83(B)), (.39(G), .37(B))).$$

* $\alpha = 0.2,\ A(G) = 4.5,\ A(B) = 4.3,\ \varphi(H) = 0.87,\ \varphi(L) = 0.85,\ \lambda(H) = 0.09,\ \lambda(L) = 0.02,\ Q(H) = 2,\ Q(L) = 1$

II. Households Problems without financial intermediaries

1. Time 0

$$V_0 = \max_{a^L_1, a^S_1, x_1} \beta E[V_1(a^L_1, a^S_1, x_1)]$$

$$st.\ a^L_1(\xi_1) + a^S_1(\xi_1) + x_1(\xi_1) = e_0$$

2. Time 1

$$V_1(a^L_1, a^S_1, x_1) = \max \left\{ \begin{array}{c} \max_{a^S_2, x_2, l^L_1, l^S_1} \log(c_1) + \beta E[V_2(a^L_1, a^S_1, a^S_2, x_2)], \\ \max_{a^S_2, x_2, l^L_1, l^S_1} \log(c_1) + \beta E[V_2(a^L_1, a^S_2, x_2)], \\ \max_{a^S_2, x_2, l^L_1, l^S_1} \log(c_1) + \beta E[V_2(a^S_1, a^S_2, x_2)], \\ \max_{a^S_2, x_2, l^L_1, l^S_1} \log(c_1) + \beta E[V_2(a^S_2, x_2)] \end{array} \right\}$$

$$st.\ c_1(\xi_1) + a^S_2 + x_2 + H_1(\xi_1) =$$

$$e_1 + \delta^L R^L_1(s_1) 1\{l^L_1 = 0\} a^L_1 + \delta^L l_1(a^L_1) 1\{l^L_1 = 1\} a^L_1 +$$

$$\delta^S R^S_1 1\{l^S_1 = 0\} a^S_1 + \delta^S l_1(a^S_1) 1\{l^S_1 = 1\} a^S_1 + x_1(\xi_1)$$

3. Time 2

$$V_2(a_1^L, a_1^S, a_2^S, x_2) = \max \left\{ \begin{array}{c} \max_{x_3, l_2^s} \log(c_2) + \beta E[V_3(a_1^L, a_2^S, x_3)], \\ \max_{x_3, l_2^s} \log(c_2) + \beta E[V_3(a_1^L, x_3)] \end{array} \right\}$$

$$st.\ c_2 + x_3 =$$

$$\delta^L R_2^L 1\{l_1^L = 0\} a_1^L + \delta^S R_2^S 1\{l_1^S = 0\} a_1^S + \delta^S R_1^S 1\{l_2^S = 0\} a_2^S + \delta^S l_1(a_2^S) 1\{l_2^S = 1\} a_2^S + x_2$$

$$V_2(a_1^L, a_2^S, x_2) = \max \left\{ \begin{array}{c} \max_{x_3, l_2^s} \log(c_2) + \beta E[V_3(a_1^L, a_2^S, x_3)], \\ \max_{x_3, l_2^s} \log(c_2) + \beta E[V_3(a_1^L, x_3)] \end{array} \right\}$$

$$st.\ c_2 + x_3 =$$

$$\delta^L R_2^L 1\{l_1^L = 0\} a_1^L + \delta^S R_1^S 1\{l_2^S = 0\} a_2^S + \delta^S l_1(a_2^S) 1\{l_2^S = 1\} a_2^S + x_2$$

$$V_2(a_1^S, a_2^S, x_2) = \max \left\{ \max_{x_3, l_2^s} \log(c_2) + \beta E[V_3(a_2^S, x_3)], \max_{x_3, l_2^s} \log(c_2) + \beta E[V_3(x_3)] \right\}$$

$$st.\ c_2 + x_3 =$$

$$\delta^S R_2^S 1\{l_1^S = 0\} a_1^S + \delta^S R_1^S 1\{l_2^S = 0\} a_2^S + \delta^S l_1(a_2^S) 1\{l_2^S = 1\} a_2^S + x_2$$

$$V_2(a_2^S, x_2) = \max \left\{ \max_{x_3, l_2^s} \log(c_2) + \beta E[V_3(a_2^S, x_3)], \max_{x_3, l_2^s} \log(c_2) + \beta E[V_3(x_3)] \right\}$$

$$st.\ c_2 + x_3 = \delta^S R_1^S 1\{l_2^S = 0\} a_2^S + \delta^S l_1(a_2^S) 1\{l_2^S = 1\} a_2^S + x_2$$

4. Time 3

$$V_3(a_1^L, a_2^S, x_3) = \max \log(c_3)$$

$$st.\ c_3 = \delta^L R_3^L 1\{l_1^L = 0\} a_1^L + \delta^S R_2^S 1\{l_2^S = 0\} a_2^S + x_3$$

$$V_3(a_1^L, x_3) = \max \log(c_3)$$

$$st.\ c_3 = \delta^L R_3^L 1\{l_1^L = 0\} a_1^L + x_3$$

$$V_3(a_2^S, x_3) = \max \log(c_3)$$

$$st.\ c_3 = \delta^S R_2^S 1\{l_2^S = 0\} a_2^S + x_3$$

$$V_3(x_3) = \max \log(c_3)$$

$$st.\ c_3 = x_3$$

III. Solutions for Households Problems without financial intermediaries

1. More Assumptions for Calculations

- For final results by hand calculations, use the following figures whenever arriving a dead end,

$$\beta = 0.98, e_1 = 1.75, h_1 = 2.5, p = 0.2$$

- Surviving endowment at time 0, $e_0 > 0.75 = h_1 - e_1$

2. Optimal Choices

- $((a_1^L, a_1^S, x_1), \{(c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^N), (c_2, x_3, l_2^S), (c_3)\},$
$\{(c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^S), (c_2, x_3, l_2^S), (c_3)\})$

(1) $e_0 > 1.969 \Rightarrow$ long-term assets(a_1^L) only, No liquidation, No corner solution($a_2^S > 0$) at time 1 shock.

$t = 0,\ (a_1^L, a_1^S, x_1) = (e_0, 0, 0)$

(No expenditure shock)

$t = 1,\ (c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^N) = (0.5952 + 0.4286e_0, 1.1548 + 0.2514e_0, 0, 0, 0; h_1^N),$

$t = 2,\ (c_2, x_3, l_2^S) = (0.6532 + 0.4705e_0, 0.2475 + 0.1356e_0, 0)$

$t = 3,\ (c_3) = (0.6401 + 0.4611e_0)$

(Expenditure shock)

$t = 1,\ (c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^S) = (-0.2551+0.4286e_0, -0.4949+0.2514e_0, 0, 0, 0; h_1^S),$

$t = 2,\ (c_2, x_3, l_2^S) = (-0.2799 + 0.4705e_0, -0.1061 + 0.1356e_0, 0)$

$t = 3,\ (c_3) = (-0.2744 + 0.4611e_0)$

(2) $1.969 > e_0 > 1.149 \Rightarrow$ long-term assets(a_1^L) only, No liquidation, Corner solution($a_2^S = 0$) at time 1 shock.

$t = 0,\ (a_1^L, a_1^S, x_1) = (e_0, 0, 0)$

(No expenditure shock)

$t = 1,\ (c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^N) = (0.5952 + 0.4286e_0, 1.1548 + 0.2514e_0, 0, 0, 0; h_1^N),$

$t = 2,\ (c_2, x_3, l_2^S) = (0.6532 + 0.4705e_0, 0.2475 + 0.1356e_0, 0)$

$t = 3,\ (c_3) = (0.6401 + 0.4611e_0)$

(Expenditure shock)

$t=1$, $(c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^S) = (-0.75 + 0.68e_0, 0, 0, 0, 0; h_1^S)$,

$t=2$, $(c_2, x_3, l_2^S) = (0.3283e_0, 0.0817e_0, 0)$

$t=3$, $(c_3) = (0.3217e_0)$

(3) $1.149 > e_0 > 1.072 \Rightarrow$ short-term assets(a_1^S) only, No Liquidation, Corner solution($a_2^S = 0$) at time 1 shock.

$t=0$, $(a_1^L, a_1^S, x_1) = (0, e_0, 0)$

(No expenditure shock)

$t=1$, $(c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^N) = (0.5952 + 0.3685e_0, 1.1548 + 0.4115e_0, 0, 0, 0; h_1^N)$,

$t=2$, $(c_2, x_3, l_2^S) = (0.6532 + 0.4045e_0, 0.2475 + 0.2565e_0, 0)$

$t=3$, $(c_3) = (0.6401 + 0.3964e_0)$

(Expenditure shock)

$t=1$, $(c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^S) = (-0.75 + 0.78e_0, 0, 0, 0, 0; h_1^S)$,

$t=2$, $(c_2, x_3, l_2^S) = (0.1717e_0, 0.1683e_0, 0)$

$t=3$, $(c_3) = (0.1683e_0)$

(4) $1.072 > e_0 > 0.75 \Rightarrow$ long-term assets and cash(a_1^L & x_1), No liquidation, Corner solution($a_2^S = 0$) at time 1 shock.

$t=0$, $(a_1^L, a_1^S, x_1) = (a_1^{L*}, 0, x_1^*)$

- $a_1^{L*} = \frac{-(2.0972+0.0568e_0)+\sqrt{(2.0972+0.0568e_0)^2+0.7691(e_0^2+e_0-1.3125)}}{0.8846}$

- $x_1^* = \frac{(2.0972+0.9414e_0)-\sqrt{(2.0972+0.0568e_0)^2+0.7691(e_0^2+e_0-1.3125)}}{0.8846}$

- ex) $(e_0 = .75 \Rightarrow a_1^{L*} = 0, x_1^* = .75), (e_0 = 1.00 \Rightarrow a_1^{L*} = .135, x_1^* = .865), (e_0 = 1.072 \Rightarrow a_1^{L*} = .177, x_1^* = .895)$,

(No expenditure shock)

$t = 1$, $(c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^N) = (0.5952 + 0.4286a_1^{L*} + 0.3401x_1^*, 1.1548 + 0.2514a_1^{L*} + 0.6599x_1^*, 0, 0, 0; h_1^N)$,

$t = 2$, $(c_2, x_3, l_2^S) = (0.6532 + 0.4705a_1^{L*} + 0.3732x_1^*, 0.2475 + 0.1356a_1^{L*} + 0.1415x_1^*, 0)$

$t = 3$, $(c_3) = (0.2475 + 0.3756a_1^{L*} + 0.1415x_1^*)$

(Expenditure shock)

$t = 1$, $(c_1, a_2^S, x_2, l_1^L, l_1^S; h_1^S) = (-0.75 + 0.68a_1^{L*} + x_1^*, 0, 0, 0, 0; h_1^S)$,

$t = 2$, $(c_2, x_3, l_2^S) = (0.3238a_1^{L*}, 0.0817a_1^{L*}, 0)$

$t = 3$, $(c_3) = (0.3217a_1^{L*})$

(4) $e_0 < 0.75 \Rightarrow$ arbitrary

APPENDIX B

TABLES AND FIGURES

Table I. Parameter Values

Parameters	Value	Note
Macro economic parameters:		
A	$\{A^G, A^L\} = \{4.0, 3.0 \sim 3.9\}$	A^G : TFP for **"Good"** states A^B : TFP for "**Bad**" states
α	0.2	Capital share
β	$[\underline{\beta}, \overline{\beta}] = [0.8, 0.99]$	Discount factor for households
β^b	0.99	Discount factor for banker
γ	$0.85 \sim 0.95$	Banker's loan collection skill
Capital quality parameters:		
$\lambda(\omega)$	$-0.3\cos(\pi\omega^{0.55}) + 0.45$	$1-$ depreciation rate
$q(\omega)$	$2\sin(\pi(\omega - 0.25)^3) + 0.5$	Cost of capital goods production
η	0.2	Capital goods maturity factor
Distributional parameters:		
$F_g(h)$	$\int \frac{(h-\underline{\beta})^\kappa}{(1-\underline{\beta})^\kappa}$	Households distribution over time preference
	where $\kappa = 1.2$ for "**Normal**" states, $\kappa = 1.08 \sim 1.17$ for "**Shock**" states	
$f_\phi(\omega)$	$0.05\cos(0.5\pi\omega) + 0.965$, $0.05\sin(0.5\pi(\omega - 1)) + 1.025$	Capital quality shock for "**Low**" states Capital quality shock for "**High**" states

Table II. Interval of Capital Quality for $\omega_t(n)$, and Efficient Capital Quality of $\omega_t^{(\cdot,n)*}$

	$\omega_t(1)$	$\omega_t(2)$	$\omega_t(3)$
Low	$[0.0967^*, 0.5536)$	$[0.5536^*, 0.9077)$	$[0.9077^*, 1]$
High	$[0.0989^*, 0.5556)$	$[0.5556^*, 0.9063)$	$[0.9063^*, 1]$

Table III. Banker's Revenues from n Period Loans $a^{(n)}$

	$a^{(1)}$	$a^{(2)}$		$a^{(3)}$		
	1^{st}	1^{st}	2^{nd}	1^{st}	2^{nd}	3^{rd}
Good & Low	1.2571	0.8624	0.7236	0.7150	0.6286	0.5661
Good & High	1.2497	0.8608	0.7226	0.7154	0.6290	0.5665
Bad & Low	1.2412	0.8515	0.7213	0.7059	0.6266	0.5656
Bad & High	1.2338	0.8499	0.7203	0.7063	0.6270	0.5660

Table IV. Banker's j^{th} Period Liquidation Values of n Period Loans $a^{(j,n)}$

	$a^{(1,2)}$	$a^{(1,3)}$	$a^{(2,3)}$
Low	0.9922	0.9941	0.5305
High	0.9881	0.9926	0.5300

Table V. Transition Matrices for Normal Economy

states of economy			states of technology			states of liquidity		
	Good	Bad		Low	High		Normal	Shock
Good	3/4	1/4	Low	2/3	1/3	Normal	3/5	2/5
Bad	1/2	1/2	High	1/2	1/2	Shock	1/2	1/2

Table VI. Transition Matrices for Productivity Shocked Economy

states of economy			states of technology			states of liquidity		
	Good	Bad		Low	High		Normal	Shock
Good	1/2	1/2	Low	2/3	1/3	Normal	3/5	2/5
Bad	1/4	3/4	High	1/2	1/2	Shock	1/2	1/2

Table VII. Transition Matrices for Liquidity Shocked Economy

states of economy			states of technology			states of liquidity		
	Good	Bad		Low	High		Normal	Shock
Good	3/4	1/4	Low	2/3	1/3	Normal	1/2	1/2
Bad	1/2	1/2	High	1/2	1/2	Shock	2/5	3/5

Table VIII. Optimal Choices and Related Results of Bank Portfolio Problem

	Good, Low & Normal			Bad, High & Shock		
	Normal	Productivity shocked	Liquidity shocked	Normal	Productivity shocked	Liquidity shocked
Loan(1), $a^{(1)}$	5.1637	4.7791	4.7019	4.7791	3.3360	2.7105
Loan(2), $a^{(2)}$	0.9444	0.6835	0.7951	0.6835	0	0
Loan(3), $a^{(3)}$	1.0027	0.8934	0.7532	0.8934	0.6093	0
Loan total, $\sum a$	7.1108	6.3560	6.2503	6.3560	4.0482	2.7105
Deposit, D	2.1576	2.1576	1.7807	2.1273	2.1273	1.6810
interest rate, r^d	1.4592	1.4592	1.9175	1.4592	1.4592	1.9175
Total output, Y	26.7130	25.8246	25.5719	25.1784	16.5904	15.3460
Bank Revenue, R	9.6167	9.2969	9.2059	9.0654	5.9750	5.5245
Survival Rate(%)	23.72	17.12	14.71	17.12	3.60	0.90

This table shows the average statistics of which equilibrium point has strictly positive total loan. The survival rate is calculated of the number of strictly positive total loan equilibria over the total grid numbers.

Table IX. Optimal Choices and Related Results of Bank Portfolio Problem with Securitization

	Good, Low & Comportable		Bad, High & Tight	
	Normal	Liquidity shock	Normal	Liquidity shock
Loan(1), $a^{(1)}$	5.3327	5.1179	4.9975	2.7105
Loan(2), $a^{(2)}$	1.6939	1.4968	0.6476	0.9821
Loan(3), $a^{(3)}$	1.1743	0.6815	0.7485	0
Loan total, $\sum a$	7.8435	7.2963	6.8195	3.8018
Deposit, D	2.1434	1.7807	2.0628	1.6810
interest rate, r^d	1.5000	1.9175	1.5000	1.9175
Total output, Y	27.1358	26.6799	25.9563	15.4348
Bank output,Y^b	27.0803	26.6594	25.9068	15.3905
Bank Revenue, $\sum R$	11.0582	10.7868	10.2612	6.0089
Securited loan, Q	1.3093	1.2005	0.6382	0.8972
Survival Rate(%)	27.63	22.82	19.22	2.70

This table shows the average statistics of which equilibrium point has strictly positive total loan, where the bank has securities business.

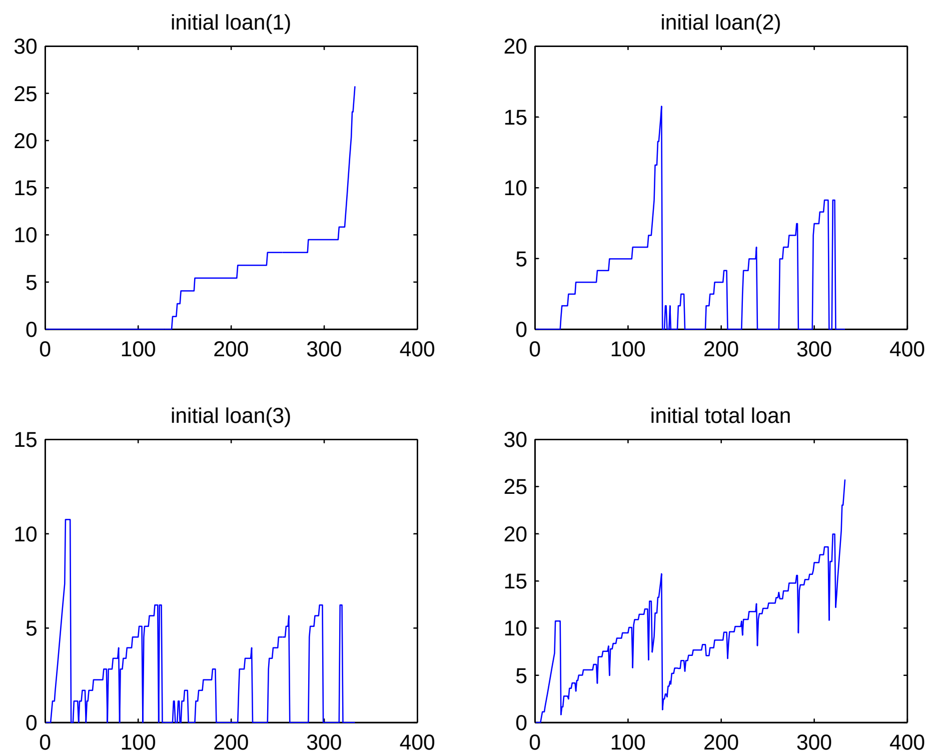

Fig. 1. Initial Loan Portfolio

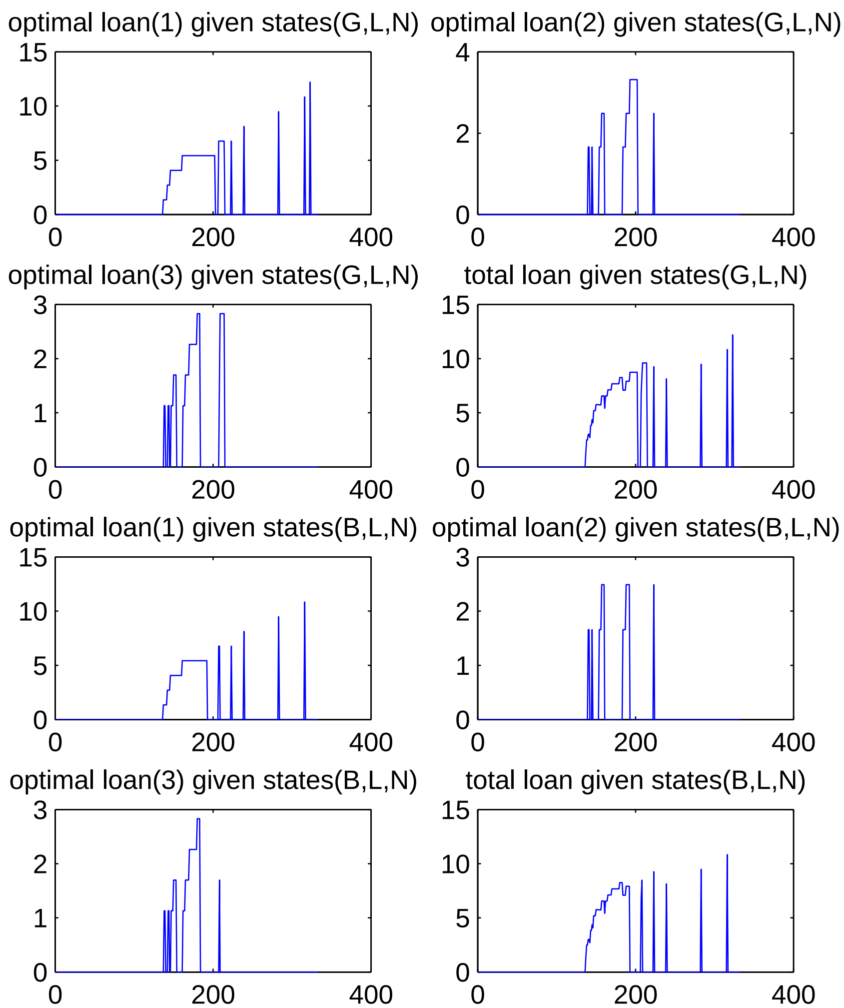

Fig. 2. Optimal Loan Portfolio Choice(1) of Normal Situation

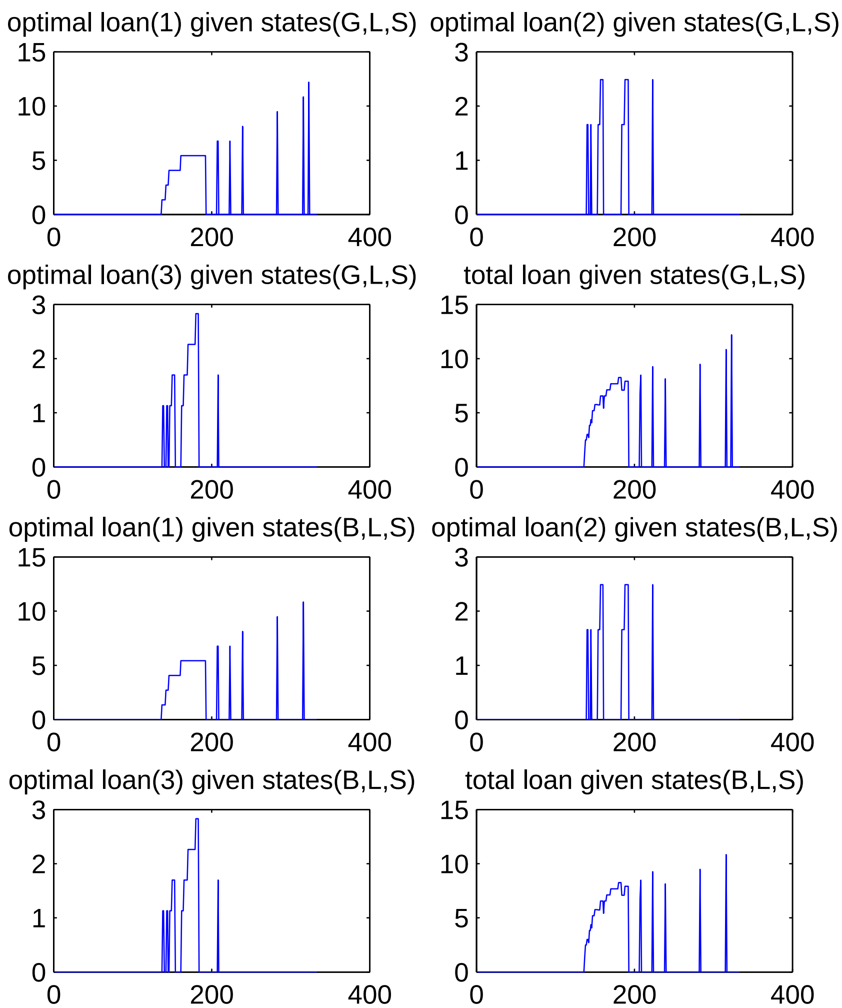

Fig. 3. Optimal Loan Portfolio Choice(2) of Normal Situation

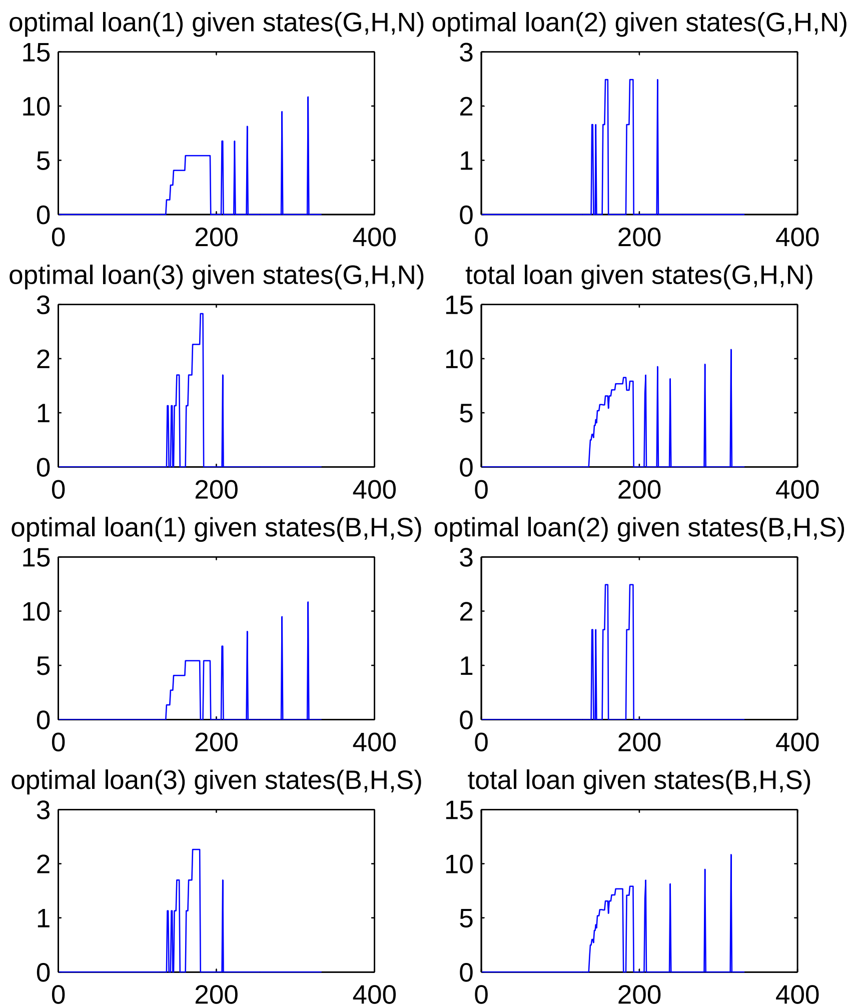

Fig. 4. Optimal Loan Portfolio Choice(3) of Normal Situation

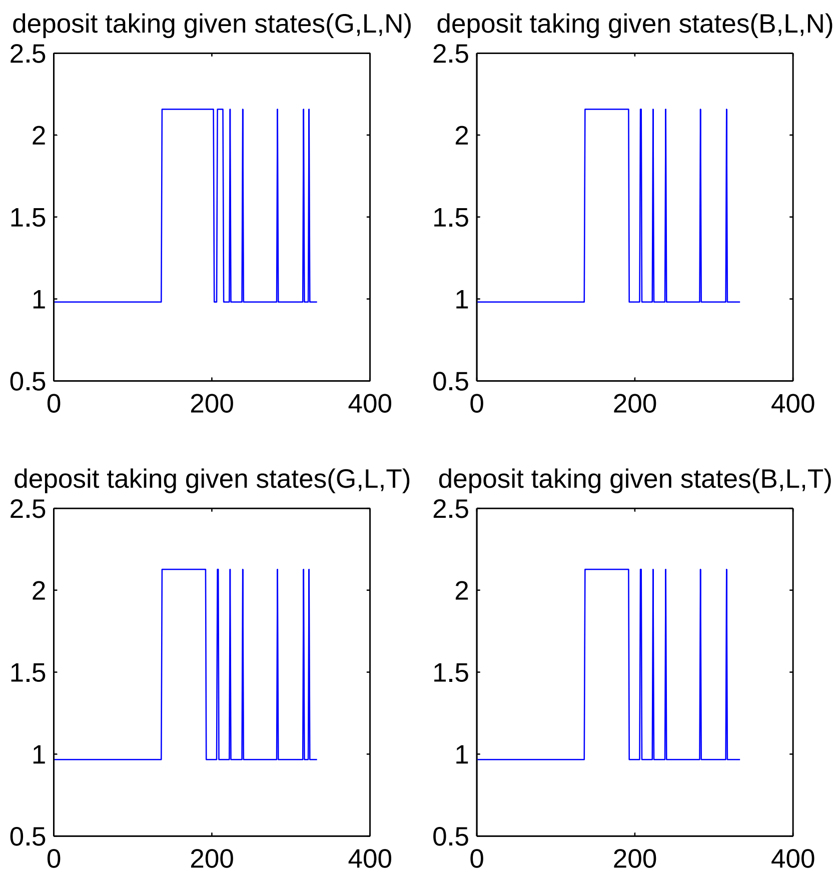

Fig. 5. Optimal Deposit Interest Rate of Normal Situation

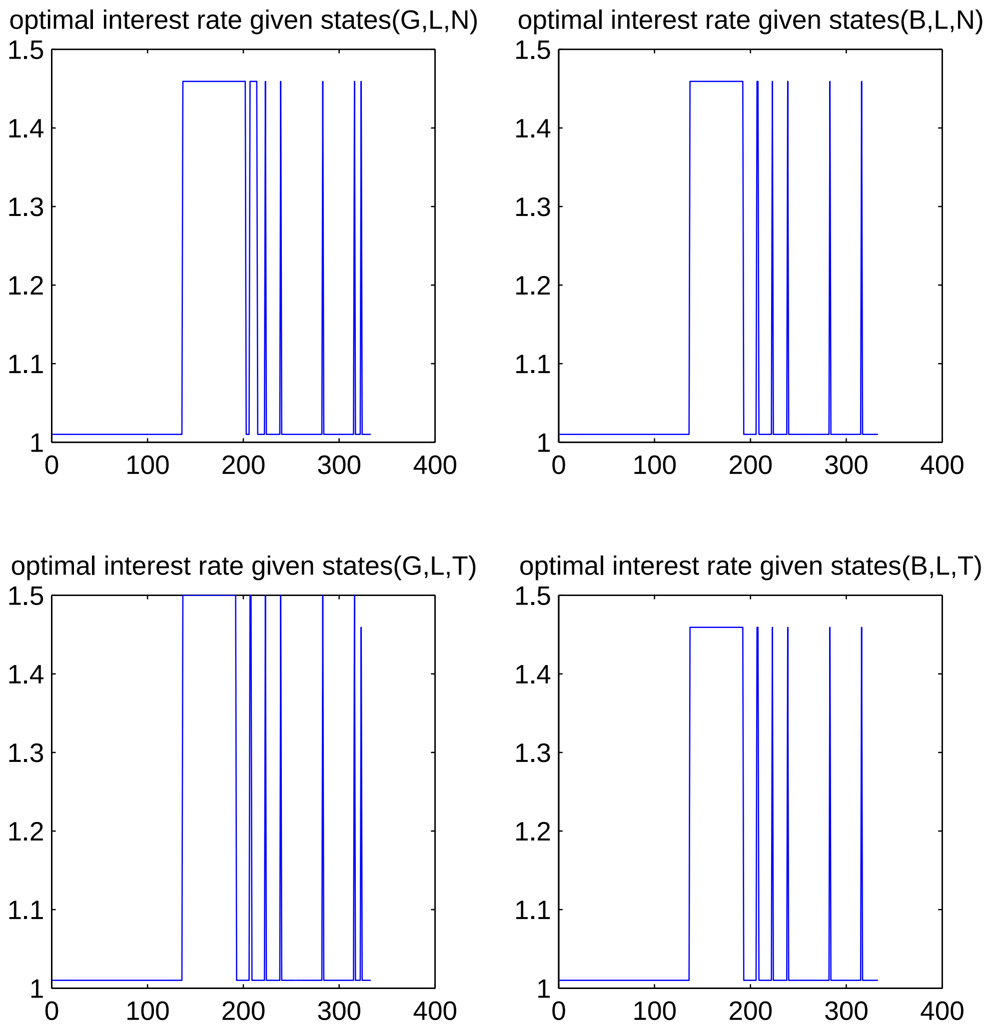

Fig. 6. Optimal Deposit Amount of Normal Situation

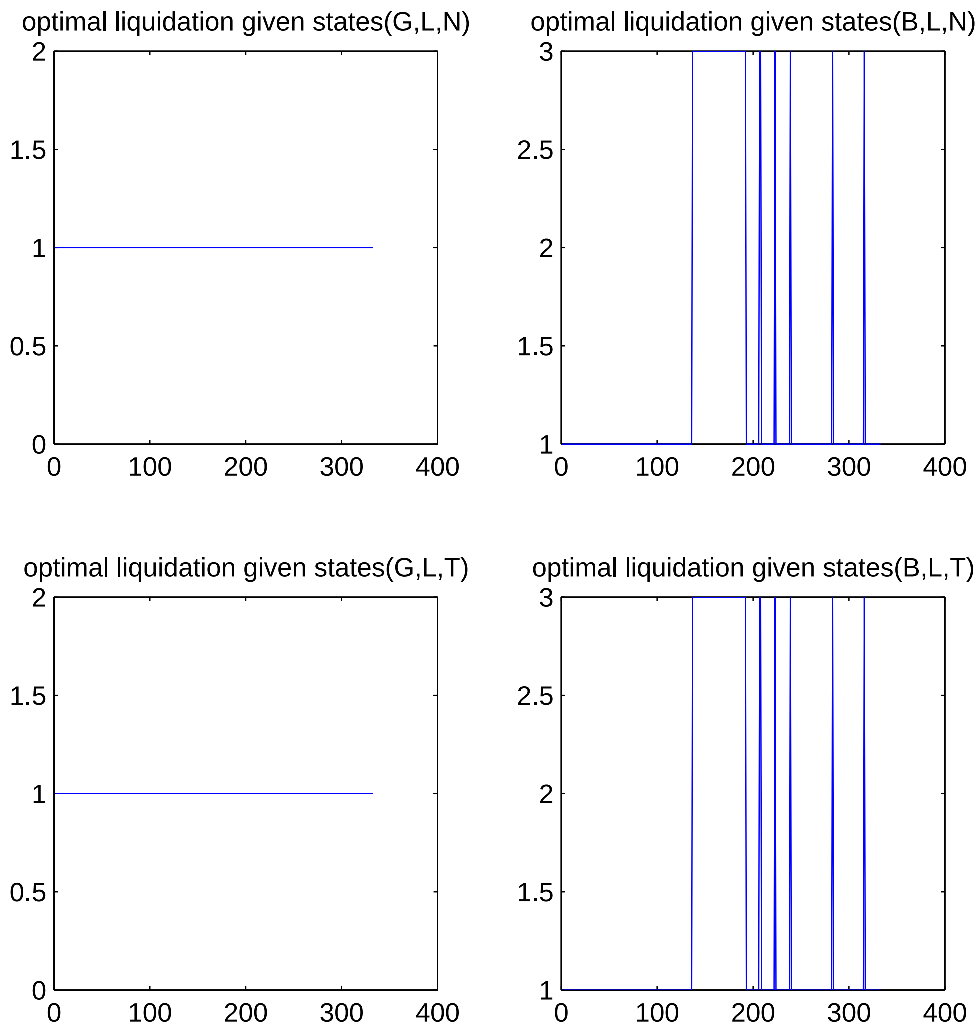

Fig. 7. Optimal Liquidation Policy of Normal Situation

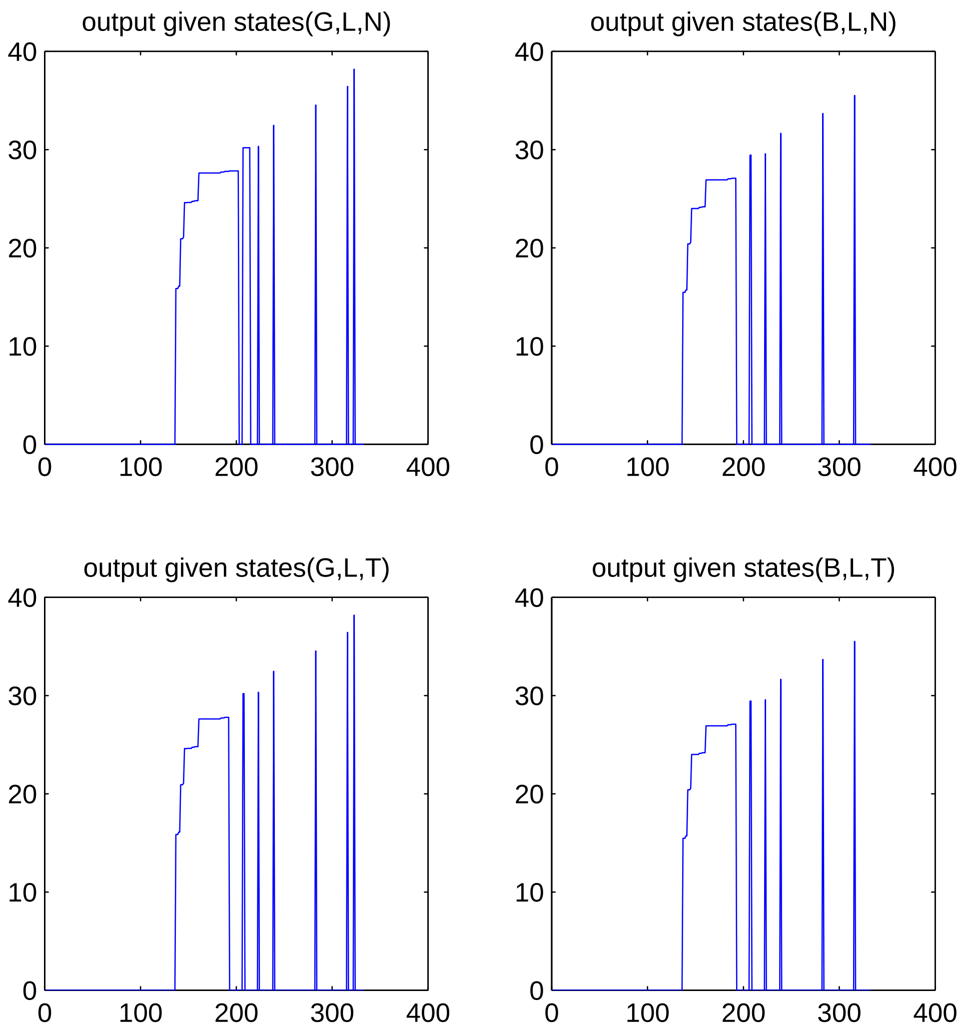

Fig. 8. Total Output of Normal Situation

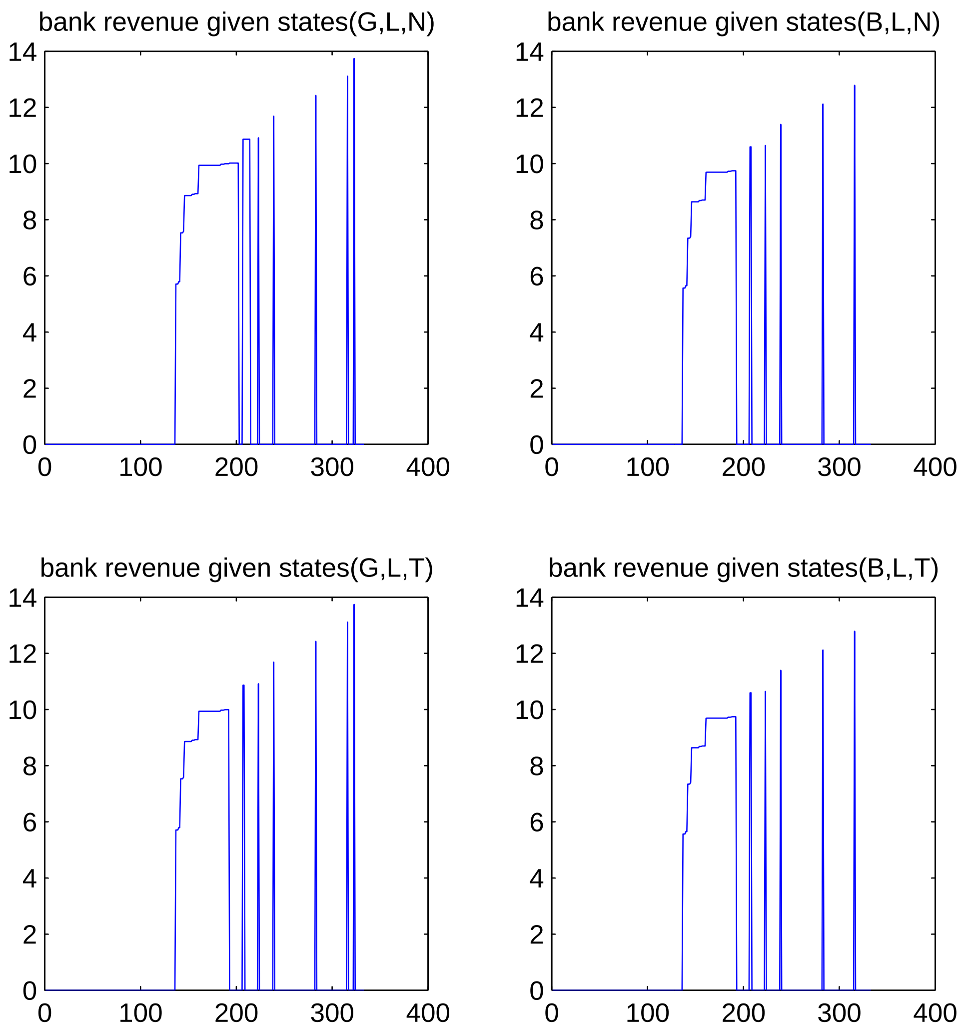

Fig. 9. Bank Revenue of Normal Situation

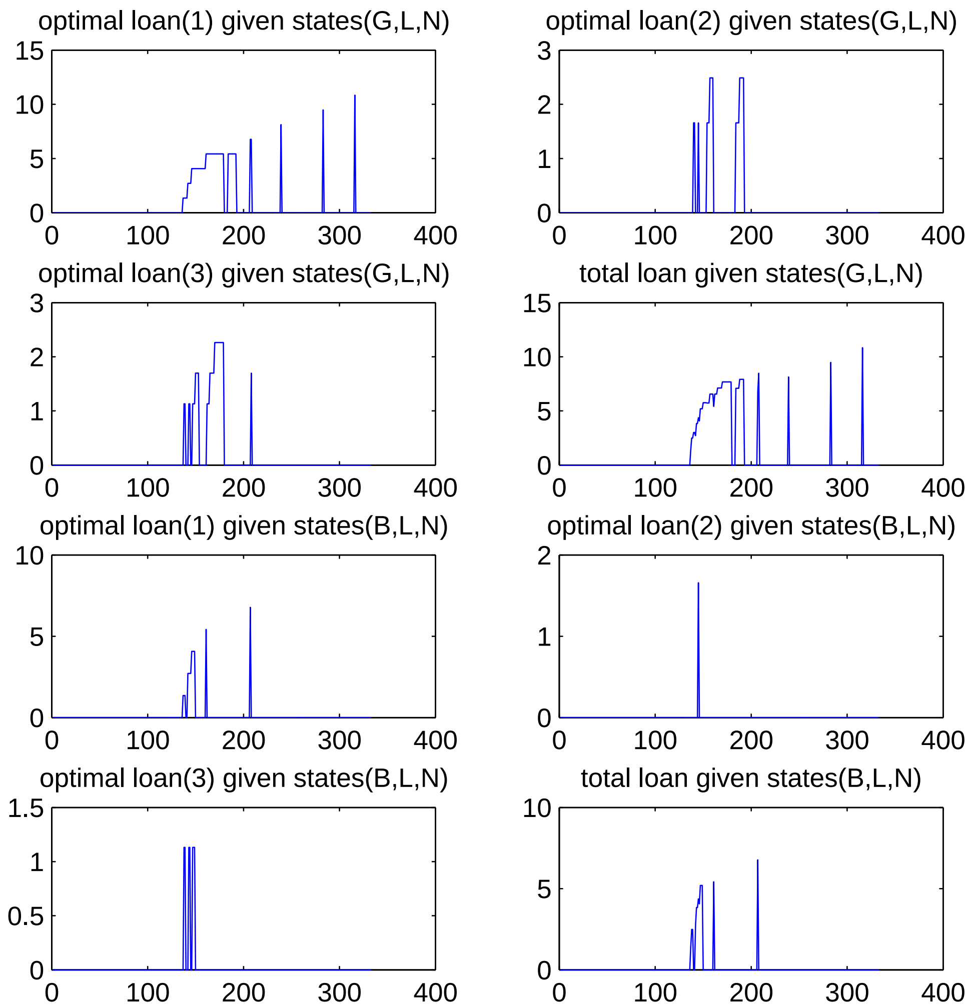

Fig. 10. Optimal Loan Portfolio Choice(1) of Productivity Shock Situation

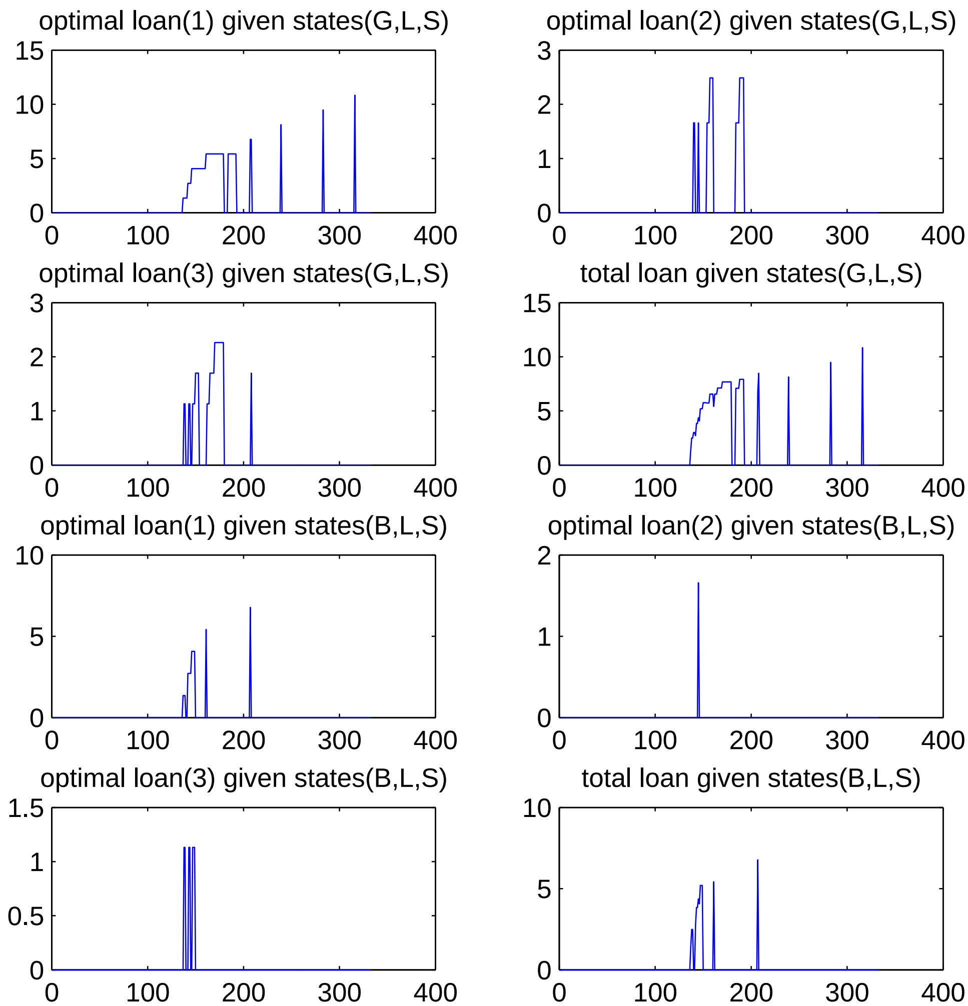

Fig. 11. Optimal Loan Portfolio Choice(2) of Productivity Shock Situation

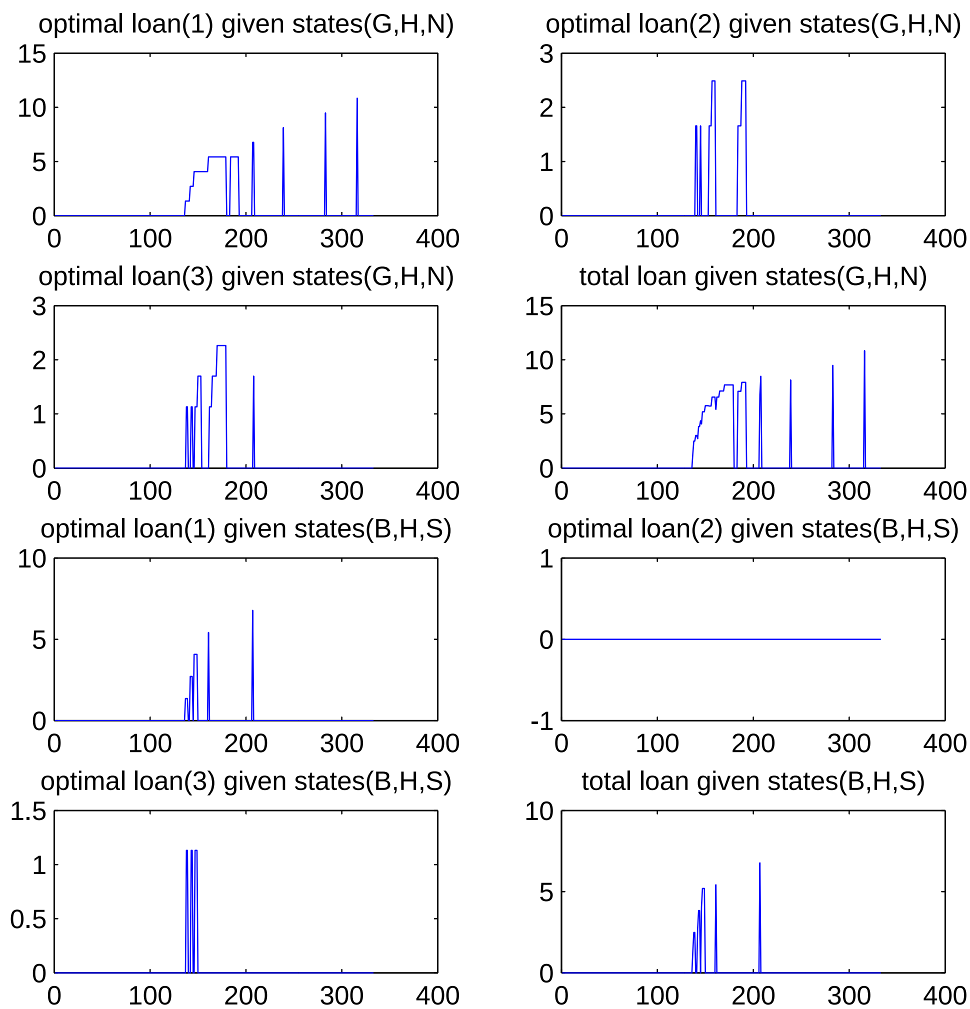

Fig. 12. Optimal Loan Portfolio Choice(3) of Productivity Shock Situation

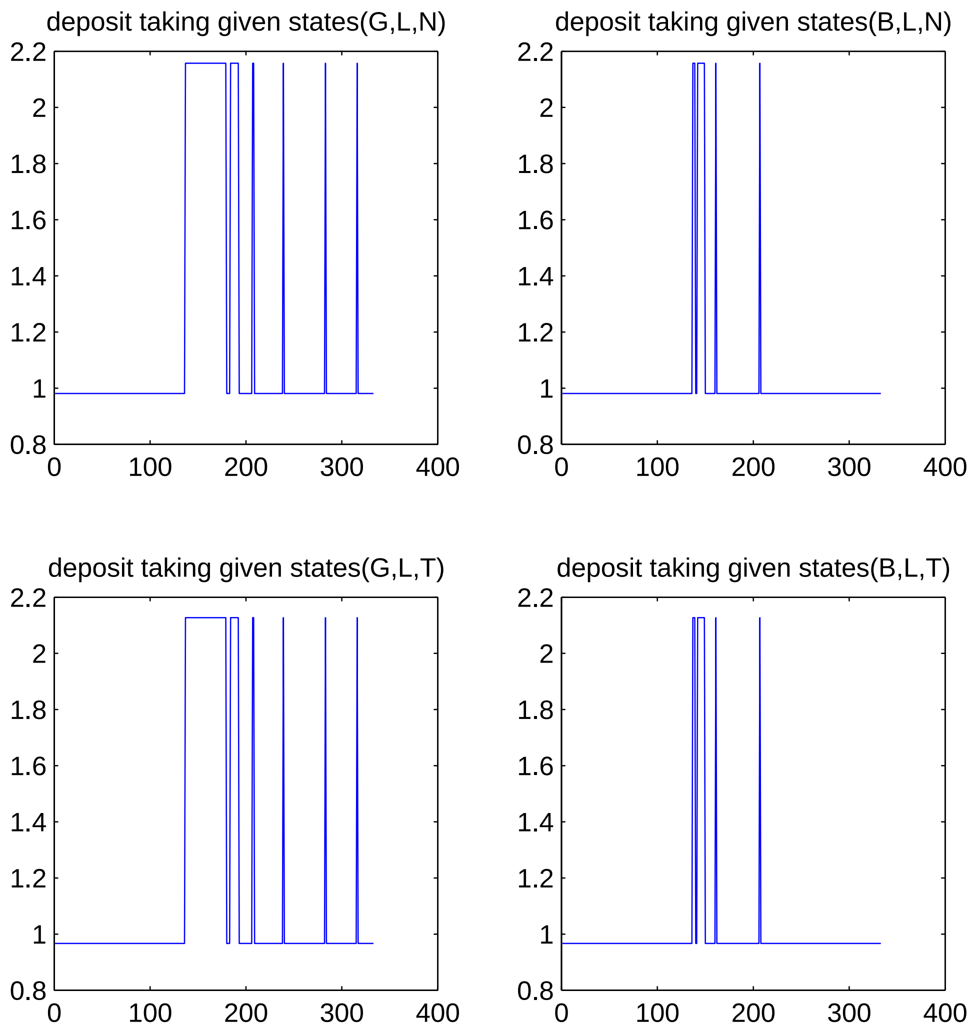

Fig. 13. Optimal Deposit Interest Rate of Productivity Shock Situation

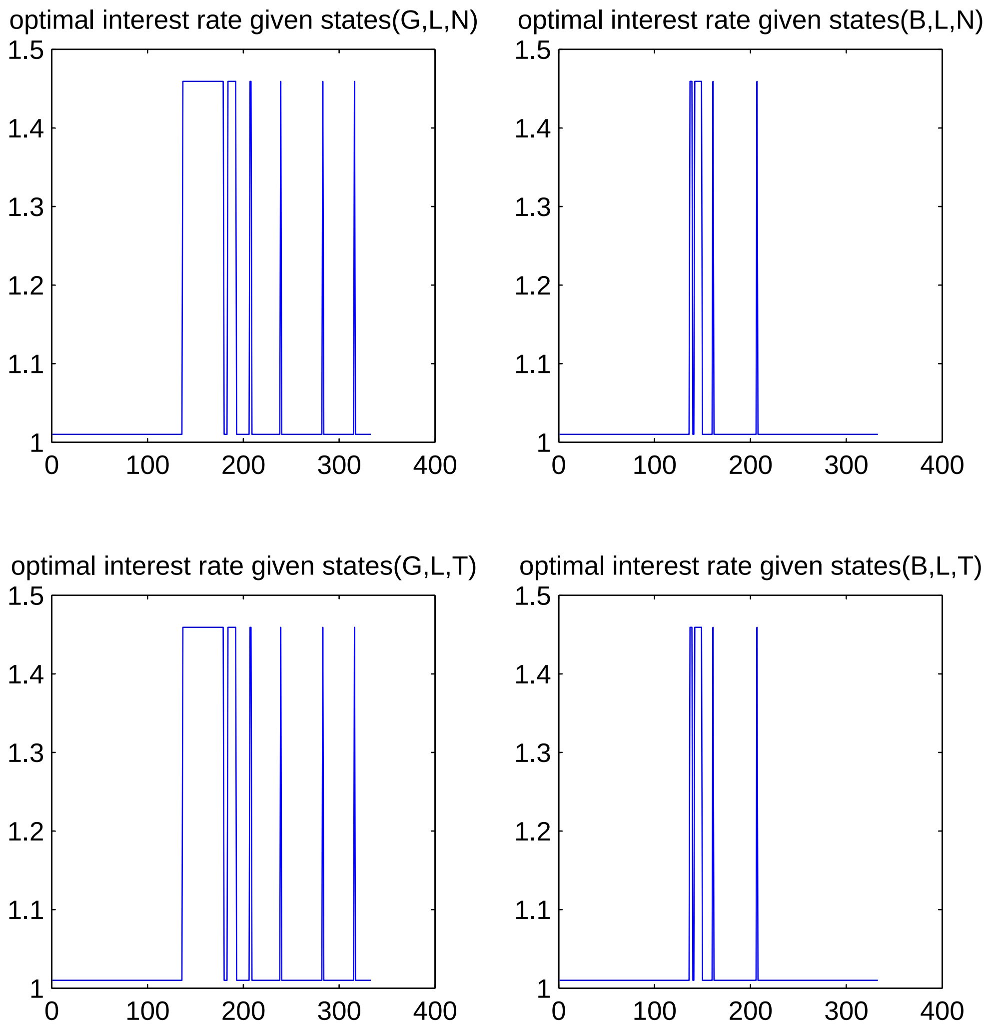

Fig. 14. Optimal Deposit Amount of Productivity Shock Situation

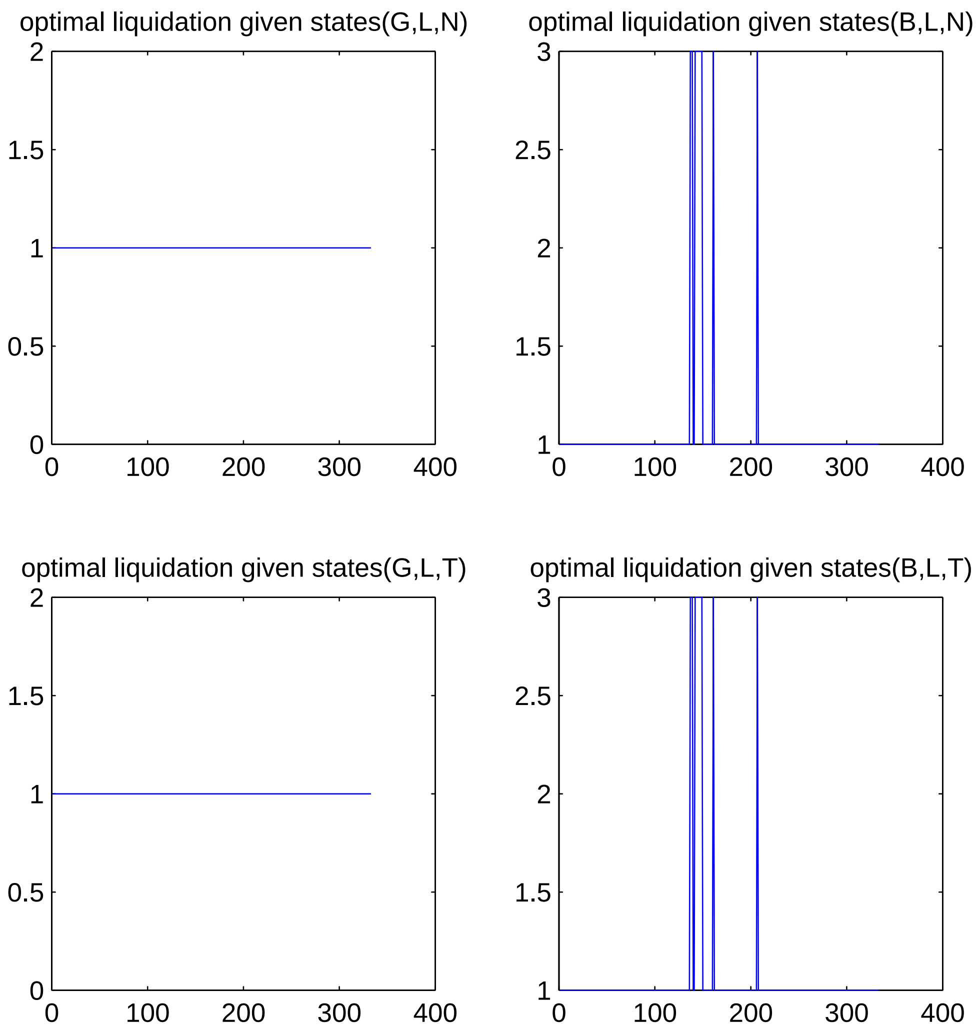

Fig. 15. Optimal Liquidation Policy of Productivity Shock Situation

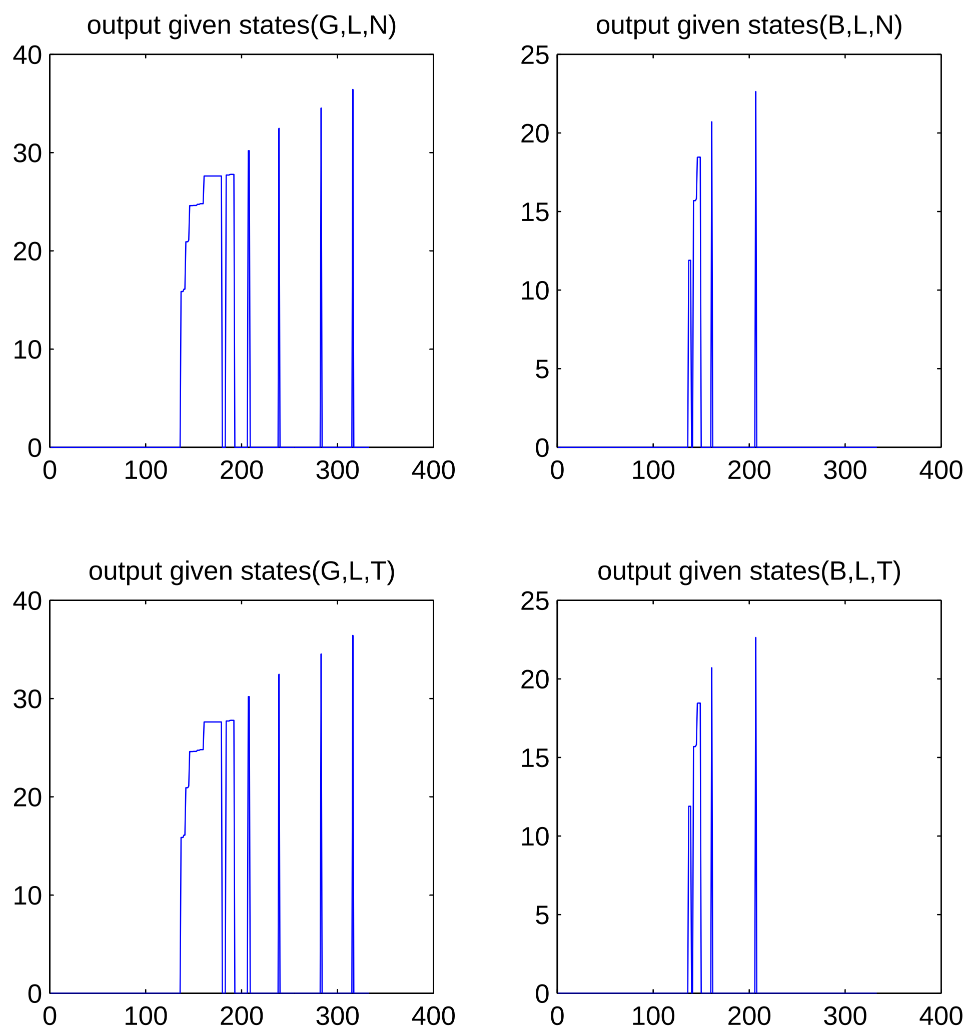

Fig. 16. Total Output of Productivity Shock Situation

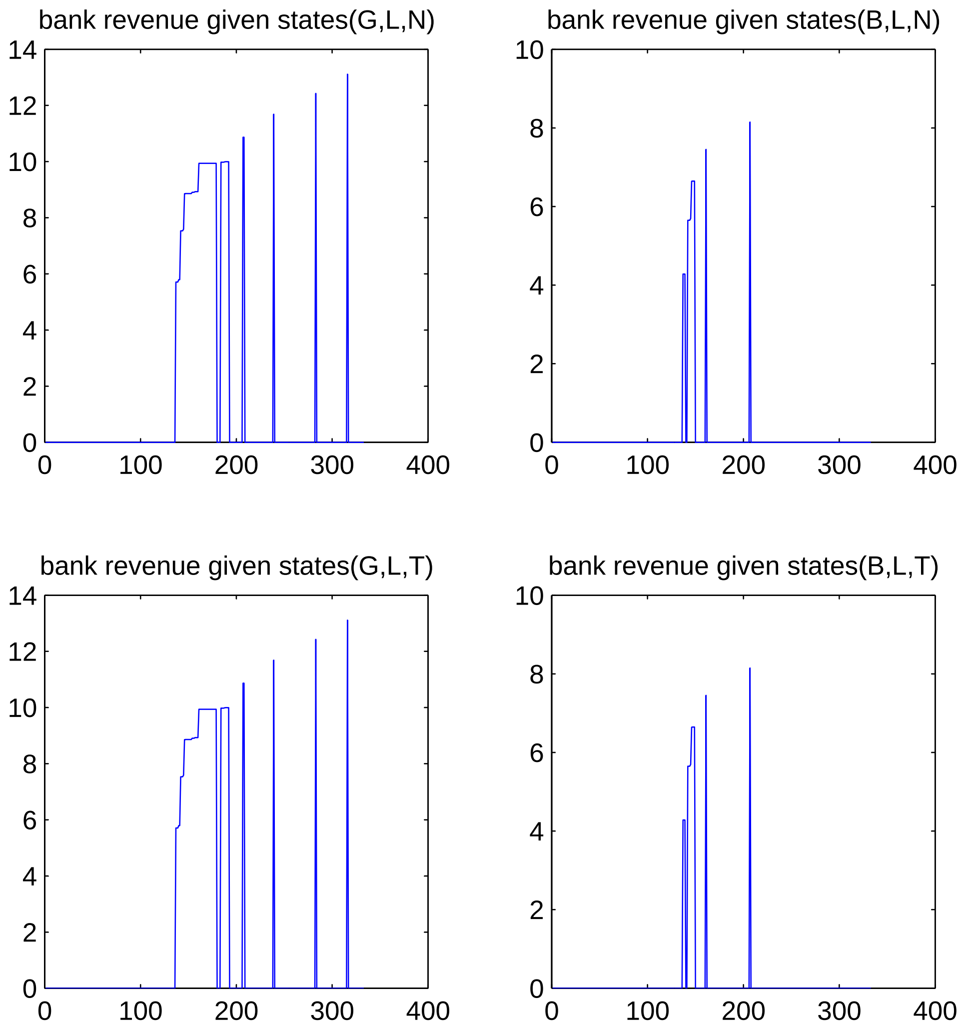

Fig. 17. Bank Revenue of Productivity Shock Situation

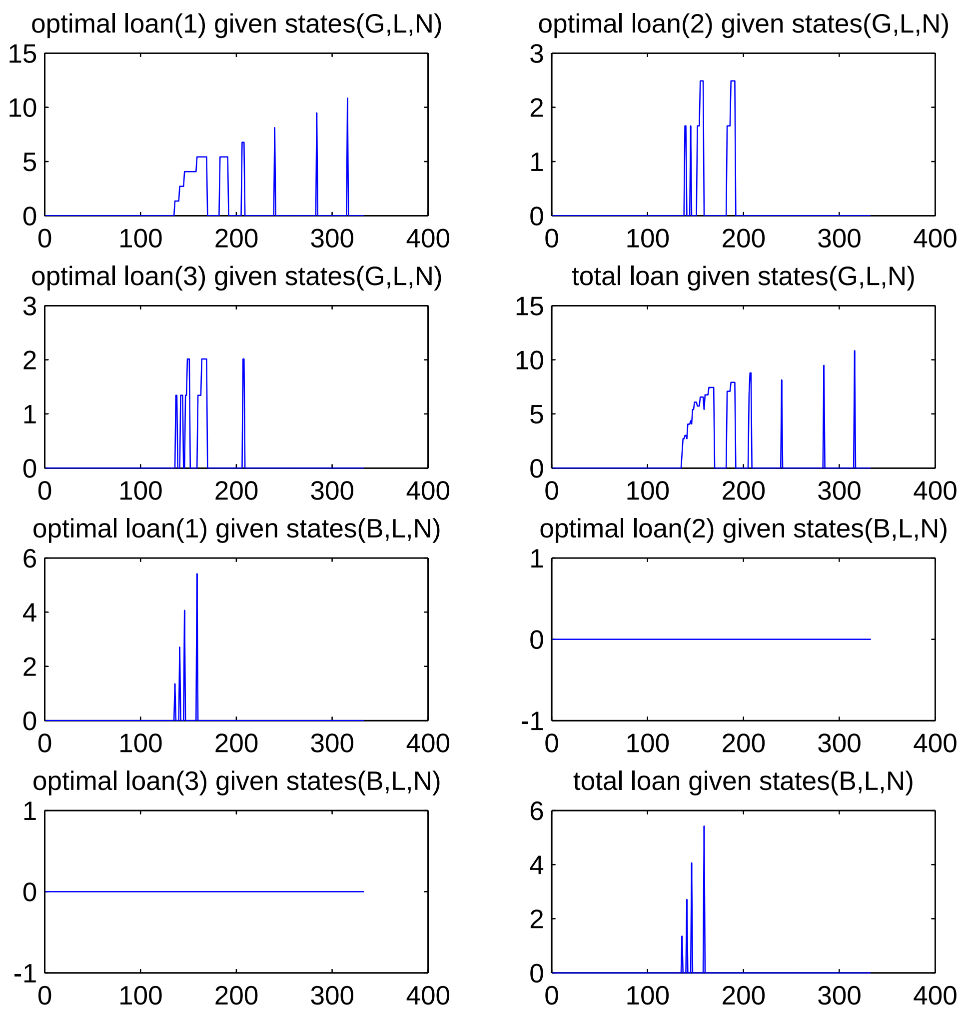

Fig. 18. Optimal Loan Portfolio Choice(1) of Liquidity Shock Situation

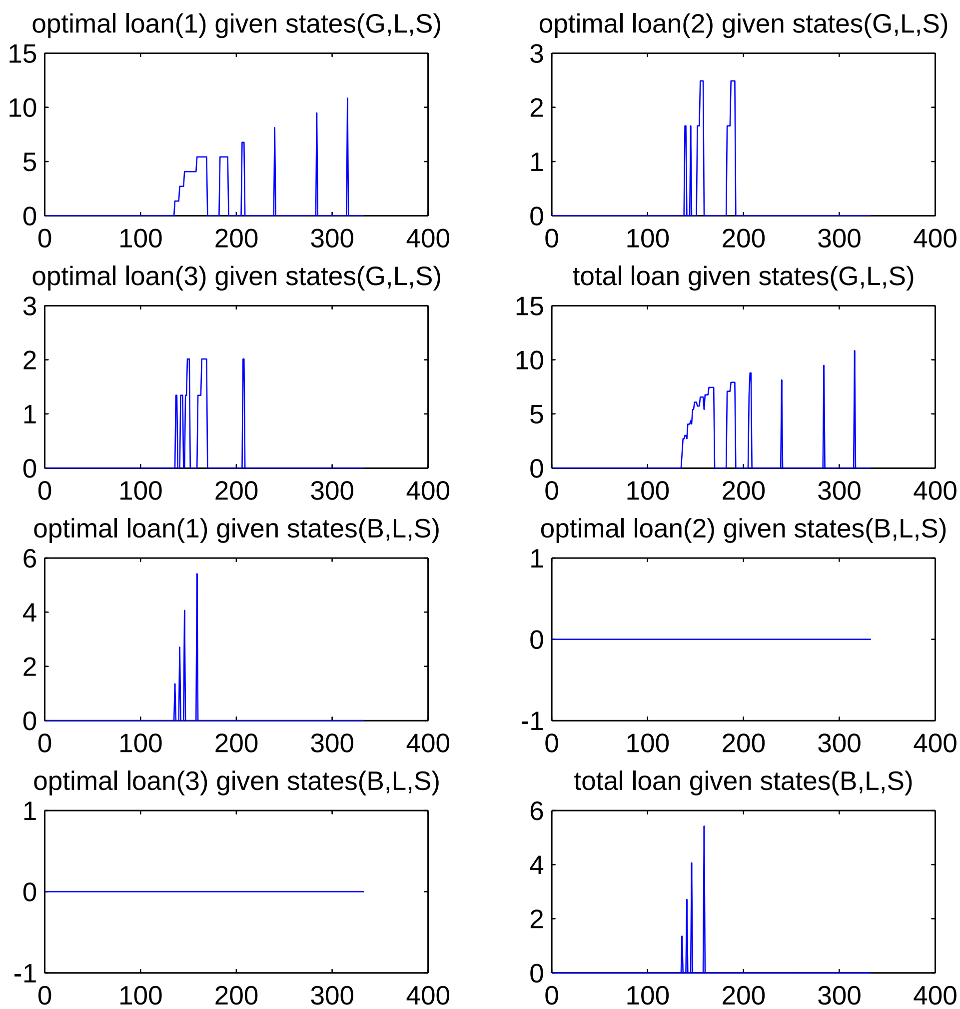

Fig. 19. Optimal Loan Portfolio Choice(2) of Liquidity Shock Situation

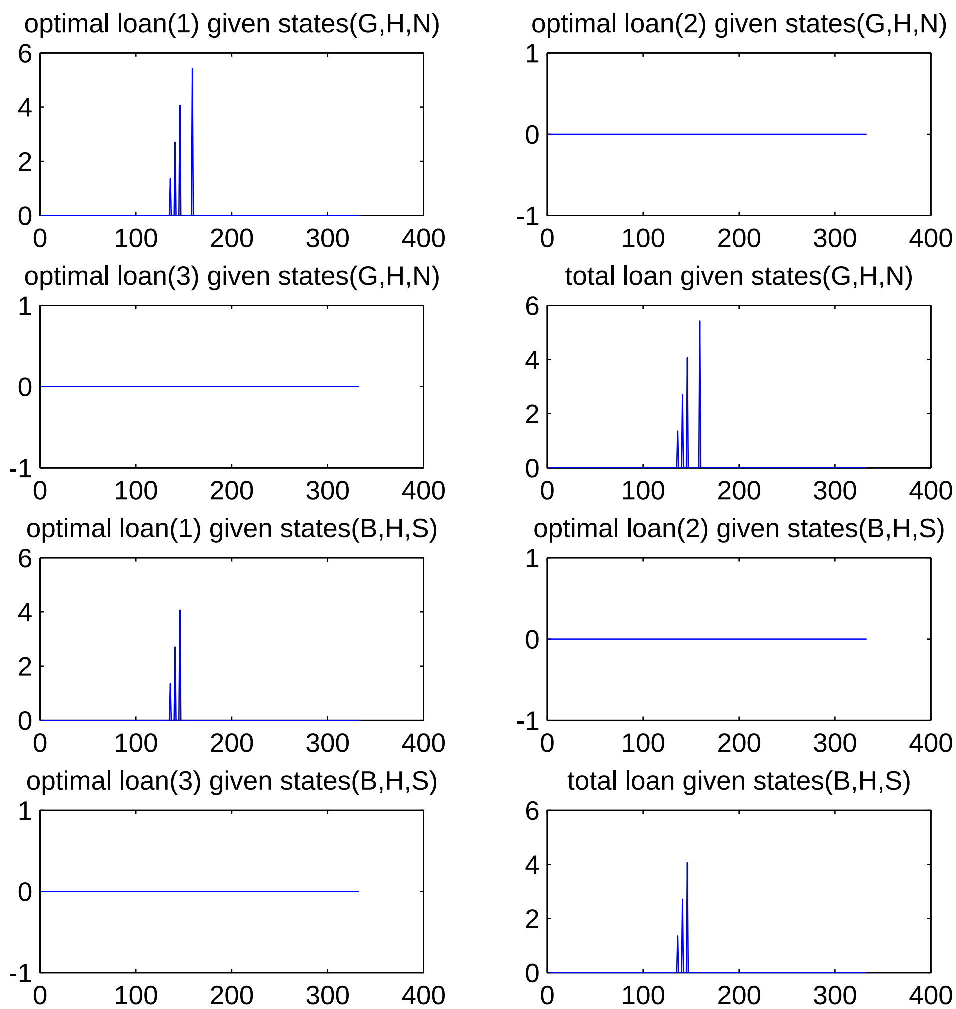

Fig. 20. Optimal Loan Portfolio Choice(3) of Liquidity Shock Situation

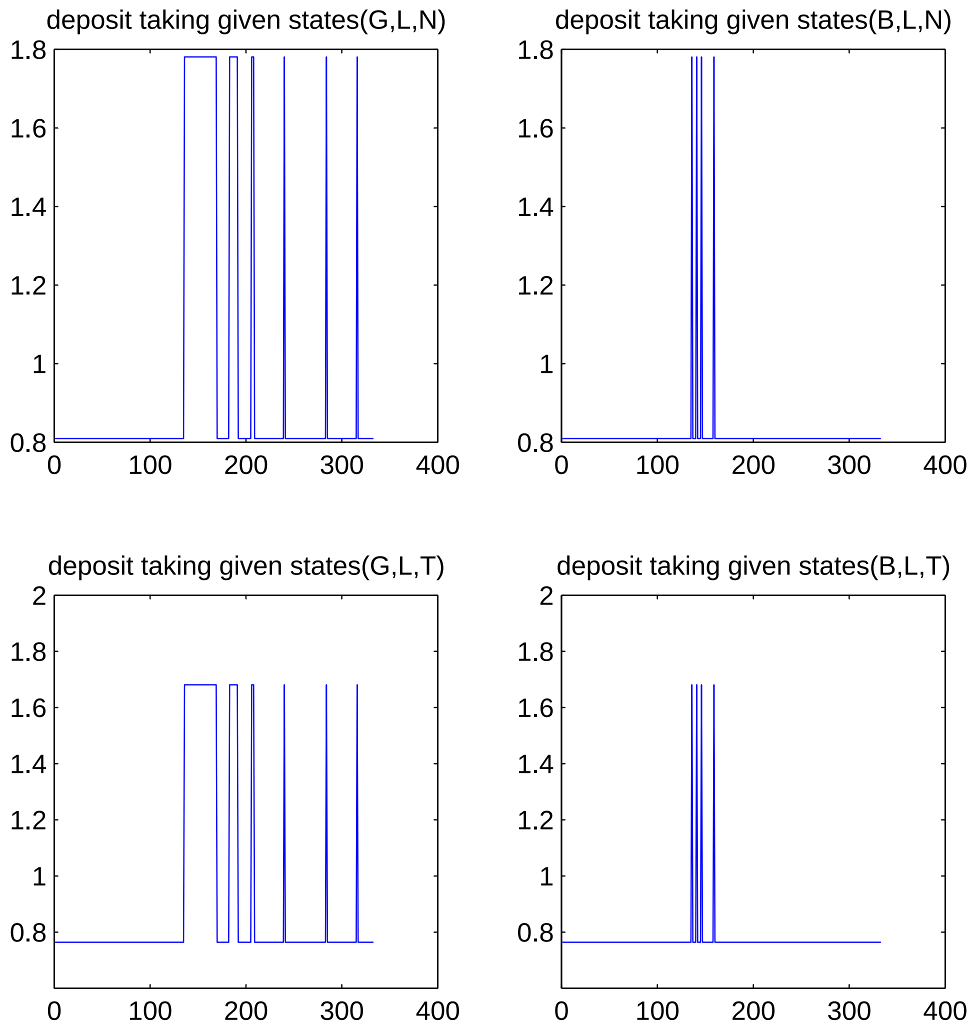

Fig. 21. Optimal Deposit Interest Rate of Liquidity Shock Situation

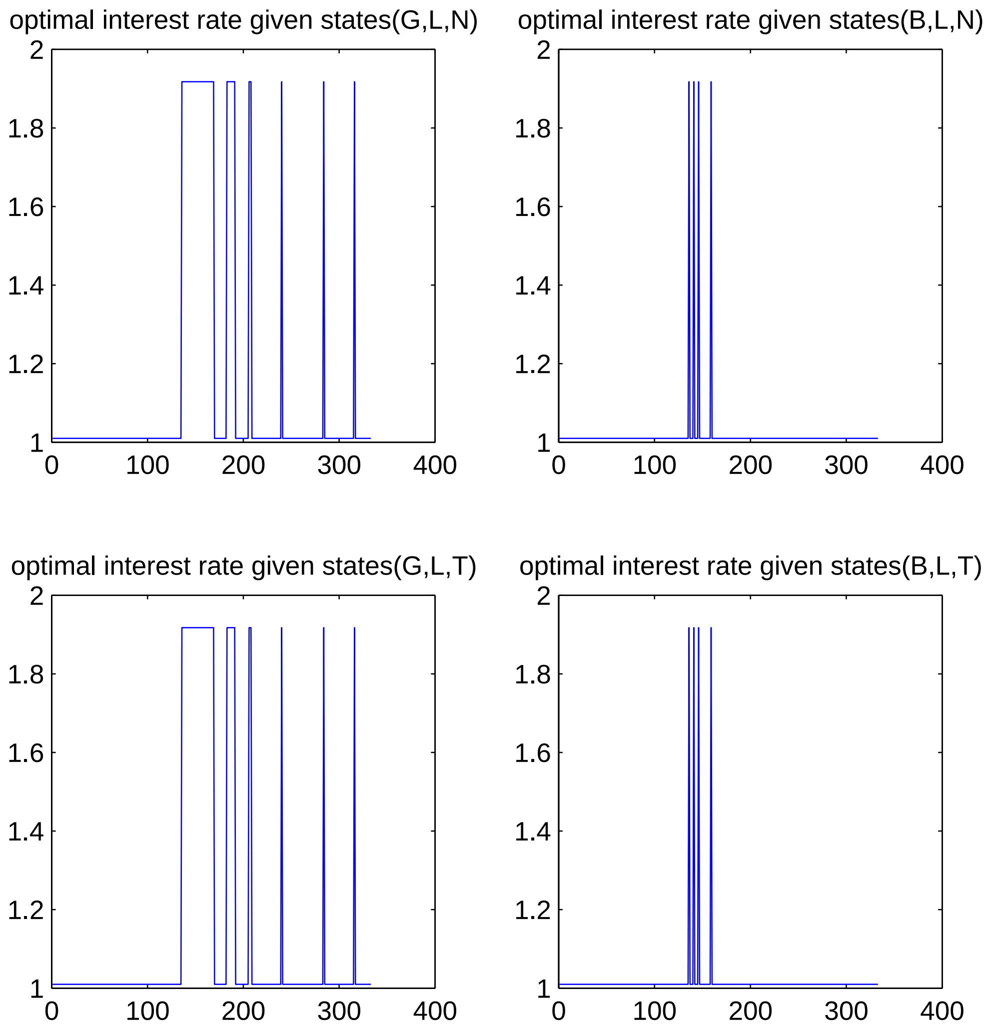

Fig. 22. Optimal Deposit Amount of Liquidity Shock Situation

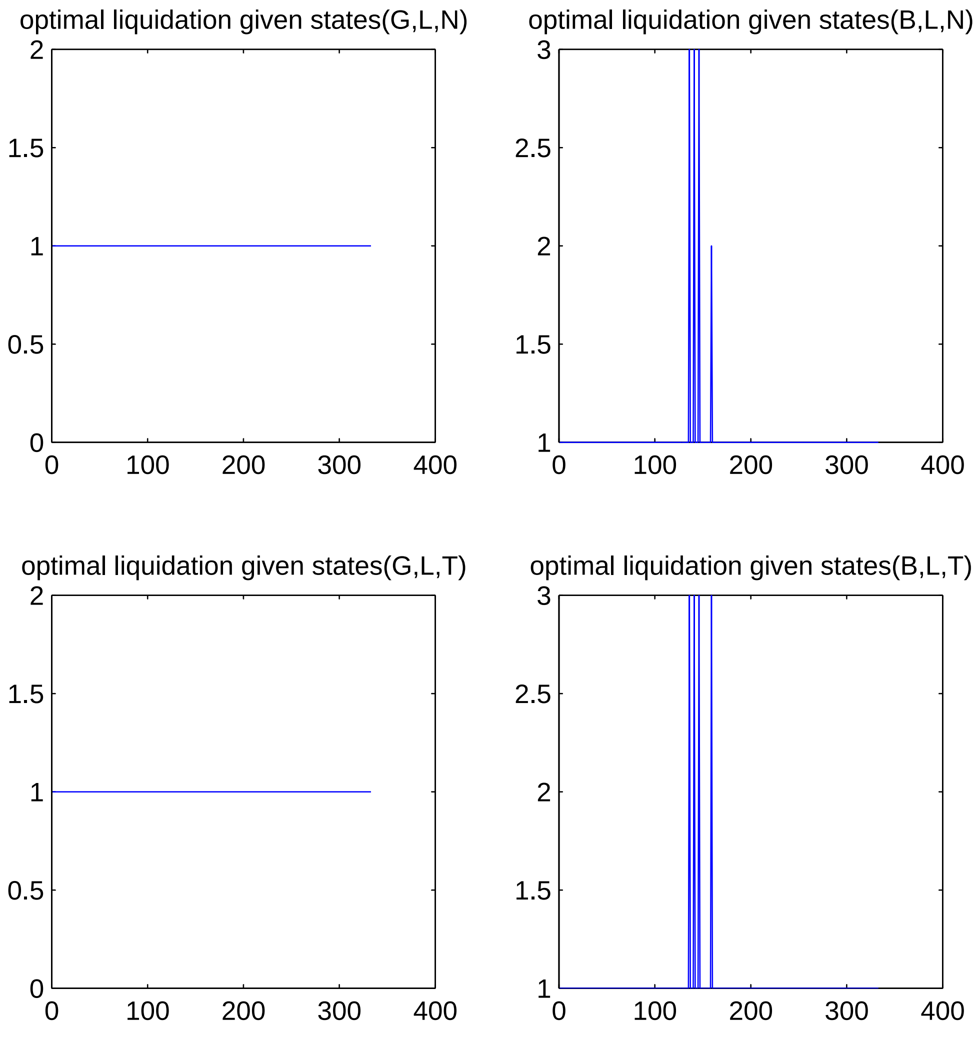

Fig. 23. Optimal Liquidation Policy of Liquidity Shock Situation

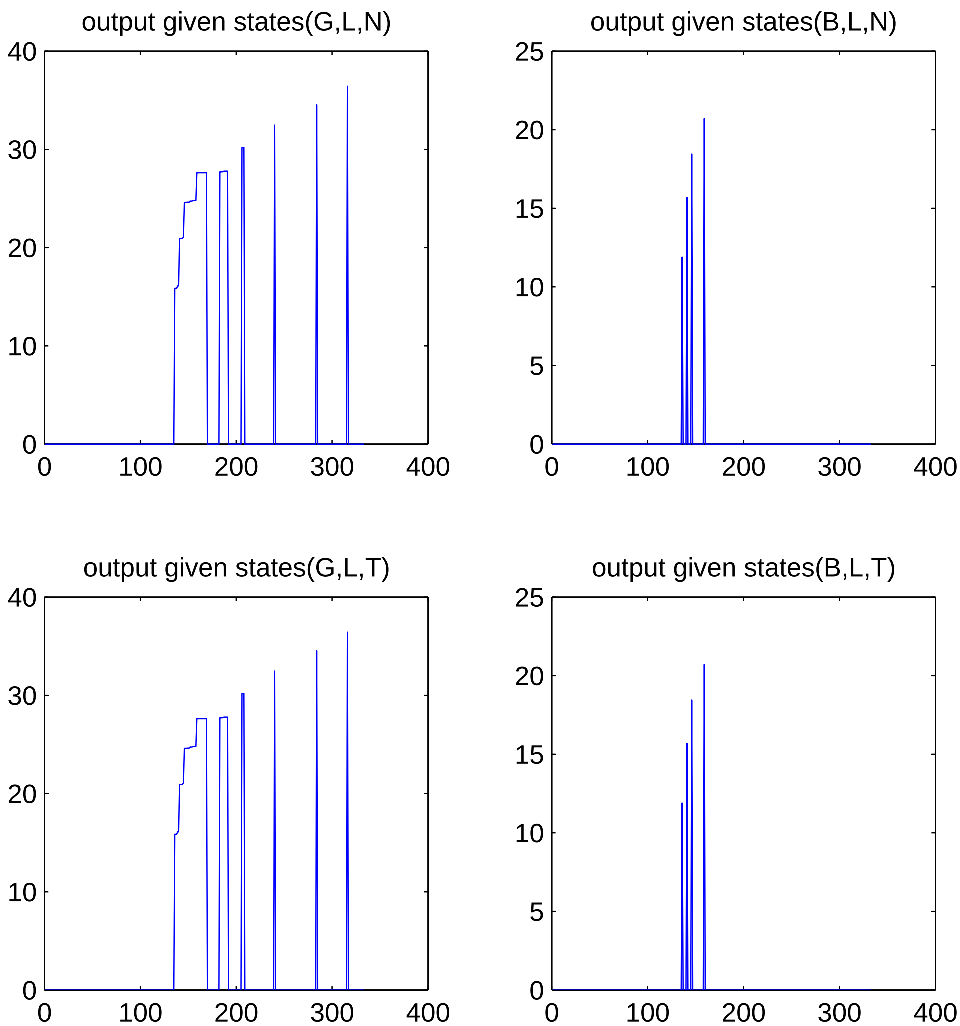

Fig. 24. Total Output of Liquidity Shock Situation

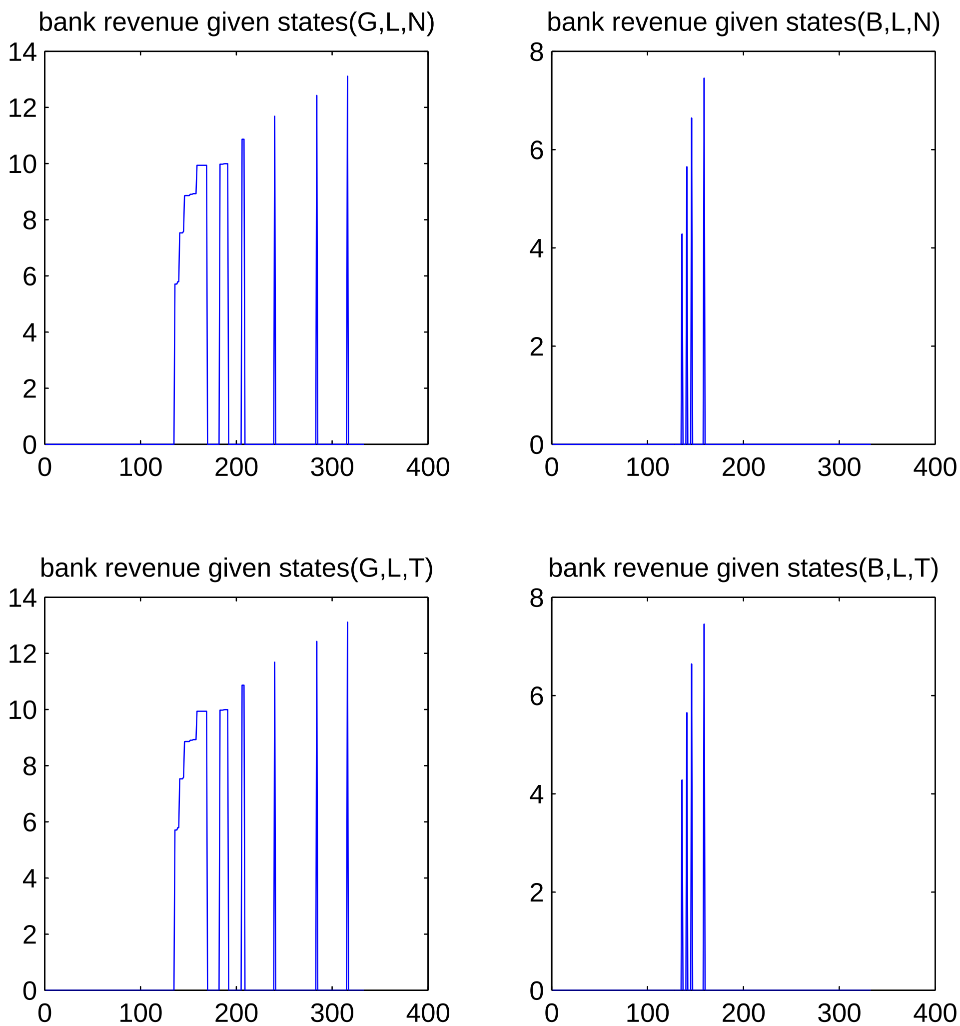

Fig. 25. Bank Revenue of Liquidity Shock Situation

VITA

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